7 Design of Beam-Columns

7.1 INTRODUCTION

Columns in a steel building are often subjected to bending moments in addition to axial compressive forces. Even when beams are connected to the column through simple connections, such as framing angles shown in Fig 7.1, they exert bending moment on the column due to eccentrically applied support reactions. Columns in moment-resisting frames, of course, are subjected to considerable bending. Members acted on simultaneously by compressive axial forces and bending moments are referred to as *beam-columns*.

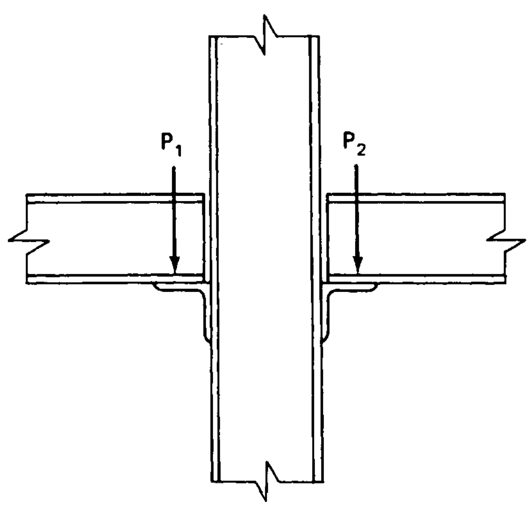


Figure 7.1

7.2 STRESSES IN BEAM-COLUMNS

When a section is subjected to an axial load P and moments M_x and M_y about its principal axes x and y, respectively (Fig. 7.2), stress at any point can be calculated approximately from the following expression:

$$f = \frac{P}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y} \tag{7.1}$$

where A is the cross-sectional area, and I_x and I_y are the moments of inertia with respect to the principal axes x and y, respectively. Equation (7.1) does not take into account the additional bending moment by the axial forces P due to the lateral deflection of the beam-column. Maximum normal stress from Eq. (7.1) is

$$f_{\max} = \frac{P}{A} + \frac{M_x c_x}{I_x} + \frac{M_y c_y}{I_y} = f_a + f_{bx} + f_{by} \tag{7.2}$$

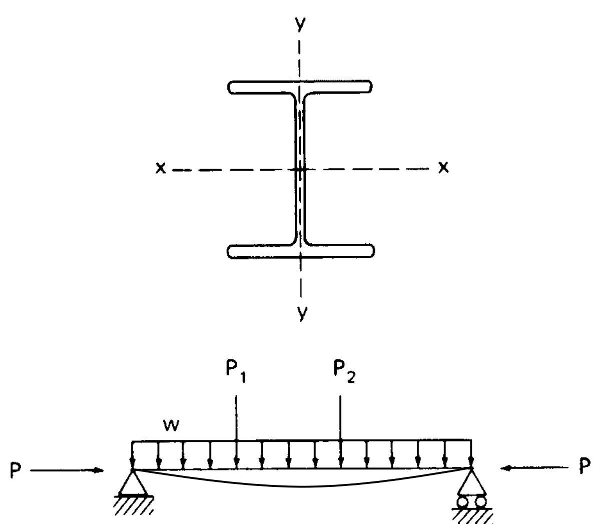


Figure 7.2

where f_a is the compressive stress due to axial load P, f_{bx} and f_{by} are the maximum bending stress due to bending moment M_x and M_y acting on the section. Noting that we have three different allowable stresses F_a, F_{bx}, and F_{by} for axial compressive stress, bending about the major axis x, and bending about the minor axis y, respectively, the following relation must be satisfied:

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \tag{7.3}$$

Until 1963, this equation was used by the ASD code for design of beam-columns. This equation is now limited to small axial compressive stresses, that is, $f_a/F_a \leq 0.15$. For larger axial compressive stresses, Eq. (7.3) has been modified to take into account the effect of additional moments produced by lateral deflections.

7.3 DESIGN OF BEAM-COLUMNS ACCORDING TO THE ASD CODE

According to ASD H1, members subjected to combined axial force and bending shall be proportioned to satisfy the following equations.

For $\frac{f_a}{F_a} \leq 0.15$:

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \tag{7.4}$$

For $\frac{f_a}{F_a} > 0.15$:

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}} \le 1.0 \tag{7.5}$$

$$\frac{f_a}{0.60 F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \le 1.0 \tag{7.6}$$

where

F_a = allowable axial compressive stress if only axial force existed

F_b = allowable compressive bending stress if only bending moment existed

f_a = actual axial compressive stress

f_b = actual compressive bending stress

F'_e = $\frac{12\pi^2 E}{23(KL_b / r_b)^2}$

= Euler stress divided by a factor of safety of 23/12 = 1.92

L_b = unbraced length in the *plane of bending*

r_b = radius of gyration in the *plane of bending*

K = effective length factor in the *plane of bending*

In Eqs. (7.4) through (7.6), subscripts x and y indicate the axis of bending. Coefficient 1./(1.- f_a/F'_e) is an amplification factor which takes care of the increased moments caused by lateral displacements. This amplification factor introduces greater conservatism compared to the older Eq. (7.4). Depending on the actual slenderness ratios, axial load, lateral loads, and end restraint conditions, this amplification factor may become excessively conservative. To offset this situation, a modification or reduction factor C_m is introduced in Eq. (7.5) whose value is less than or equal to one.

To find the value of C_m, the ASD code divides beam-columns into these categories:

1. For compression members in frames subjected to sidesway or joint translation [for example, member *AB* in the moment-resisting frame of Fig. 7.3 (a)]:

$$C_m = 0.85 \tag{7.7}$$

2. For compression members in frames braced against sidesway or joint translation and not subjected to transverse loading between their ends [for example, member *AB* in the braced frame of Fig 7.3 (b)]:

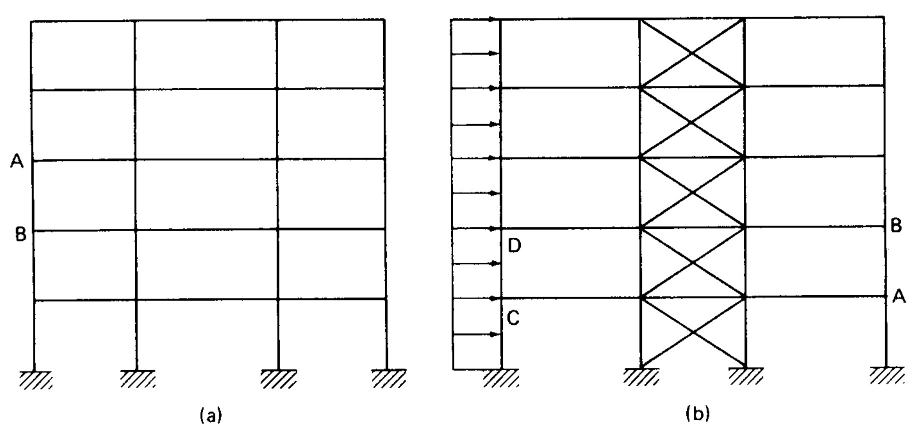


Figure 7.3 Frames with and without sidesway. (a) unbraced frame, (b) braced frame

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} \tag{7.8}$$

where M_1/M_2 is the ratio of the smaller moment to the larger moment at the ends of the unbraced length in the plane of bending. The ratio M_1/M_2 is positive when the end moments M_1 and M_2 are in the same direction (reverse curvature) and negative otherwise (single curvature). A member in single curvature in general has larger lateral displacements than a corresponding member in double curvature and consequently is subjected to larger moments and larger bending stresses. When the two end moments have the same magnitude but opposite direction ($M_1 = -M_2$), C_m becomes equal to unity.

3. For compression members in frames braced against sidesway and subjected to transverse loading [for example, member *CD* in the braced frame of Fig. 7.3(b)], the value of C_m is found from the following expression:

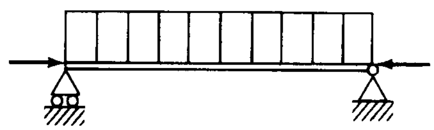
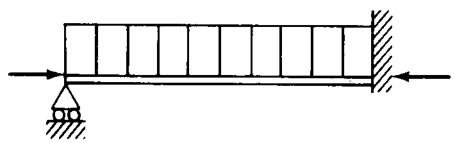
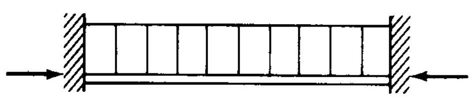
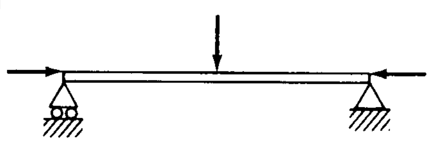
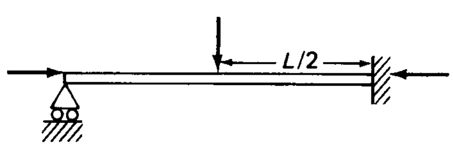
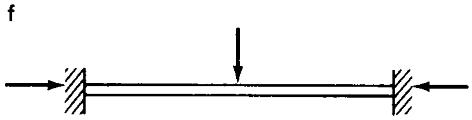
$$C_m = 1. + \psi \frac{f_a}{F_e'} \tag{7.9}$$

where ψ is a dimensionless factor whose values for several end restraint and loading conditions are given in Table 7.1. For other cases, the ASD code recommends the following values:

For members with moment restraint at the ends:

$$C_m = 0.85 \tag{7.10}$$

TABLE 7.1 REDUCTION FACTOR C_m (ASD COMMENTARY TABLE C-H1.1)

Case	ψ	C_m
a 	0	1.0
b 	−0.4	$1 - 0.4\,\frac{f_a}{F'_e}$
c 	−0.4	$1 - 0.4\,\frac{f_a}{F'_e}$
d 	−0.2	$1 - 0.2\,\frac{f_a}{F'_e}$
e 	−0.3	$1 - 0.3\,\frac{f_a}{F'_e}$
f 	−0.2	$1 - 0.2\,\frac{f_a}{F'_e}$

For members with simply supported ends:

$$C_m = 1.0 \qquad (7.11)$$

In Eqs. (7.4) — (7.6), F_a is found on the basis of the maximum slenderness ratio without regard to the plane of bending. In contrast, F'_e is calculated on the basis of the slenderness ratio in the plane of bending.

Equations (7.6) and (7.5) may be considered as yield and stability criteria, respectively. Per ASD Commentary H1, in calculating f_b in Eq. (7.6) the larger of the two end moments is used. On the other hand, when the member is subject to intermediate transverse loads, the maximum bending moment between points of supports must be used to calculate f_b in Eq. (7.5).

When bending takes place about one axis only, the term corresponding to the other axis in Eqs. (7.4) — (7.6) shall be deleted.

7.4 DESIGN OF BEAM-COLUMNS ACCORDING TO THE ASD CODE USING EQUIVALENT AXIAL COMPRESSIVE LOAD

Equations (7.4) — (7.6) cannot be readily used for design of beam-columns. These equations can be rearranged so that we need to design the beam-column for an equivalent axial compressive load. In other words, when a member is acted upon by an axial compressive force P and a bending moment M, the bending moment can be converted into an equivalent axial load P' and consequently the member needs to be designed for an equivalent axial load of $P_{eq} = P + P'$.

In the case of bending about the x-axis only, Eq. (7.5) becomes

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{(1 - f_a / F'_{ex}) F_{bx}} \leq 1.0 \qquad (7.12)$$

Substituting for $f_a = P/A$ and $f_{bx} = M_x/S_x$ and multiplying the two sides by AF_a, we find that Eq. (7.12) at the limit becomes

$$P + \frac{M_x A}{S_x}\left(\frac{F_a}{F_{bx}}\right)\left[\frac{C_{mx}}{1 - P/(AF'_{ex})}\right] = AF_a \tag{7.13}$$

Note that $P_{eq} = AF_a$ would be the design load if the member were axially loaded. Defining

$$a_x = \frac{12\pi^2 EAr_x^2}{23} \tag{7.14}$$

we can write

$$AF'_{ex} = \frac{12\pi^2 EA}{23(KL/r_x)^2} = \frac{a_x}{(KL)^2} \tag{7.15}$$

$$\frac{C_{mx}}{1 - P/(A/F'_{ex})} = \frac{a_x C_{mx}}{a_x - P(KL)^2} \tag{7.16}$$

If we denote $B_x = A/S_x$, called the bending factor, the equivalent axial compressive load will be

$$P_{eq} = P + B_x M_x C_{mx}\left(\frac{F_a}{F_{bx}}\right)\left[\frac{a_x}{a_x - P(KL)^2}\right] \tag{7.17}$$

Similarly, in general, the three Eqs. (7.4), (7.5), and (7.6) can be written in the following equivalent axial load form:

$$P_{eq} = P + P'_x + P'_y = P + B_x M_x \left(\frac{F_a}{F_{bx}} \right) + B_y M_y \left(\frac{F_a}{F_{by}} \right) \tag{7.18}$$

$$\begin{aligned} P_{eq} &= P + P'_x + P'_y = P + B_x M_x C_{mx} \left(\frac{F_a}{F_{bx}} \right) \left[\frac{a_x}{a_x - P(KL)^2} \right] \\ &\quad + B_y M_y C_{my} \left(\frac{F_a}{F_{by}} \right) \left[\frac{a_y}{a_y - P(KL)^2} \right] \end{aligned} \tag{7.19}$$

$$P_{eq} = P + P'_x + P'_y = P \left(\frac{F_a}{0.60 F_y} \right) + B_x M_x \left(\frac{F_a}{F_{bx}} \right) + B_y M_y \left(\frac{F_a}{F_{by}} \right) \tag{7.20}$$

In Eq. (7.19), K is the effective length factor and L is the actual unbraced length in the plane of bending. For W shapes tabulated in the ASD manual, the ranges of bending factors B_x and B_y are

$$0.072 \le B_x = \frac{A}{S_x} \le 0.701 \tag{7.21}$$

$$0.408 \le B_y = \frac{A}{S_y} \le 3.496 \tag{7.22}$$

Also,

$$a_x = 149{,}000 A r_x^2 \tag{7.23}$$

$$a_y = 149{,}000 A r_y^2 \tag{7.24}$$

In these equations, A is in square inches, r_x is in inches, and S_x and S_y are in cubic inches.

Design of beam-column is an iterative process. To start this process, the following rough approximation may be used for the equivalent axial load:

$$P_{eq} \approx P + B_x M_x + B_y M_y \tag{7.25}$$

For the initial trial selection, average value may be used for B_x and B_y, as summarized in Table 7.2.

Table 7.2 AVERAGE VALUES FOR BENDING FACTORS B_x AND B_y

Types of section	B_x (1/in.)	B_y (1/in.)
W14	0.18	0.50
W12	0.21	0.69
W10	0.26	0.83
W8	0.33	1.02
W6	0.46	1.83

7.5 EFFECTIVE LENGTH OF COLUMNS IN BRACED AND UNBRACED FRAMES

The end conditions for a column in a braced or unbraced frame is usually different from the end conditions for the columns shown in Fig.

6.2. For a practical method of finding the effective length or equivalent hinged-ends length for a column in a frame, the following assumptions are usually made (Galambos, 1968):

1. Columns have elastic behavior.
2. Columns are prismatic.
3. The frame is a rectangular and symmetrical structure.
4. Axial forces in girders are negligible.
5. Columns at a joint carry the end girder moments in proportion to their stiffnesses.
6. All columns attain their buckling loads simultaneously.
7. At the incipient buckling the rotation of the girder at its ends are equal and opposite.

On the basis of these assumptions, one can find the following transcendental equation for the effective length factor K in a braced frame such as the one shown in Fig. 7.3(b) (Galambos, 1968):

$$\frac{G_A G_B}{4}\left(\frac{\pi^2}{K^2}\right)+\left(\frac{G_A+G_B}{2}\right)\left(1-\frac{\pi/K}{\tan \pi/K}\right)+\frac{2}{\pi/K}\tan\frac{\pi}{2/K}=1 \tag{7.26}$$

In this equation, subscripts A and B refer to the ends of column AB and

$$G=\frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_g}{L_g}} \tag{7.27}$$

where I_c is the moment of inertia and L_c the unsupported length of a column section, I_g is the moment of inertia and L_g the unsupported length of a girder, and $\sum$ indicates a summation for all members connected to the joint under consideration and lying in the plane of buckling.

Similarly, the transcendental equation for the effective length factor K in an unbraced frame is as follows (Galambos, 1968):

$$\frac{G_A G_B (\pi / K) - 36}{6(G_A + G_B)} = \frac{\pi / K}{\tan \pi / K} \tag{7.28}$$

Equations (7.26) and (7.28) must be solved numerically. For manual design of frames, nomograms or alignment charts have been developed to be used in place of these equations. These alignment charts

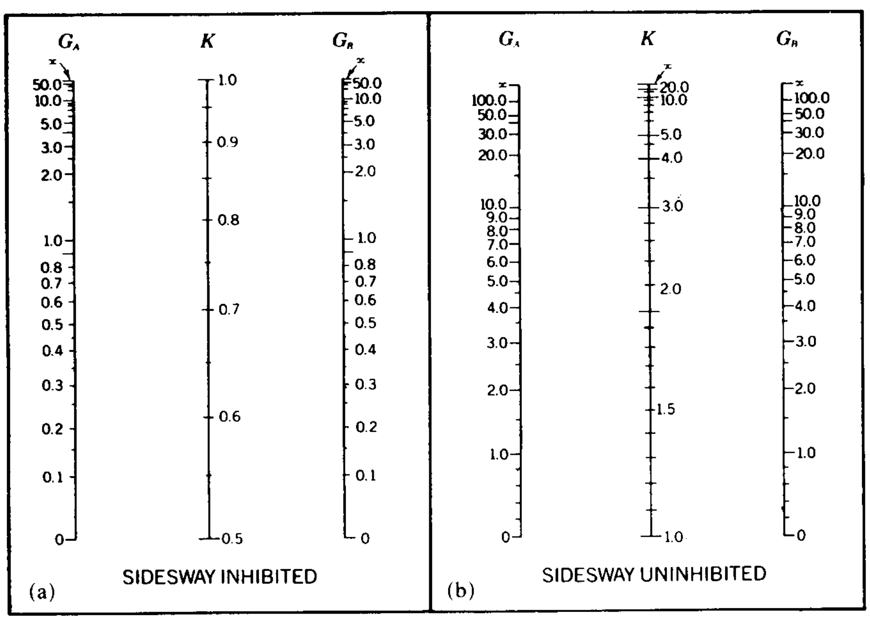


Figure 7.4 Alignment charts for the effective length factors in braced and unbraced frame (AISC, 1995). Printed by permission of the AISC

are presented in Fig. 7.4. To find the effective length factor for a column, first quantities G_A and G_B are found for the two ends of the column. By connecting the corresponding points of the alignment chart, a straight line is obtained. The intersection of this line with the *K*-line in the chart yields the value of the effective length factor.

For columns supported by a footing or foundation through a simple support, *G* is theoretically infinity, but the ASD code recommends a practical value of 10. For columns rigidly connected to a footing or foundation, a practical value of 1.0 is recommended for *G* (AISC, 1995).

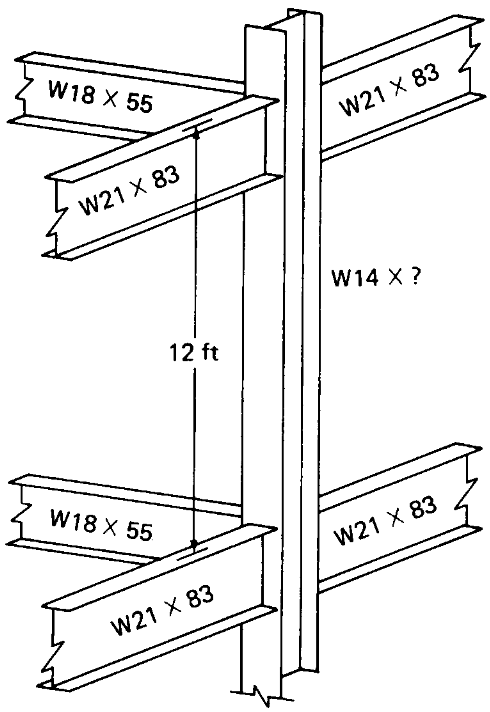
7.6 EXAMPLES OF DESIGN OF BEAM-COLUMNS ACCORDING TO THE ASD CODE

Example 1

Design the W14 steel column shown in Fig. 7.5. The column must be designed for an axial load of 400 Kips, bending moment about the major axis of 200 K-ft, and bending moment about the minor axis of 100 K-ft. The column is a member of a moment-resisting space frame (with “rigid” connections). Use A36 steel with yield stress of 36 ksi and assume the columns in tier above and below to be the same as the column shown in the figure. The average values of bending coefficients B_x and B_y for W14 sections are 0.18 1/in. and 0.5 1/in., respectively.

Solution

P = 400 K $\qquad$ M_x = 200 K-ft $\qquad$ M_y = 100 K-ft



Spacing of columns in each direction: 25 ft. **Figure 7.5**

From Eq. (7.25):

$$P_{eq} \approx 400 + 0.18(200)(12) + 0.5(100)(12) = 1432 \text{ K}$$

In general, to design a beam-column, several trials are necessary. For the first trial one may assume a certain value for the allowable axial compressive stress F_a. In this example, let us assume

$$F_a = 0.50F_y = 18 \text{ ksi}$$

In that case, a rough estimate for the cross-sectional area will be

$$A = \frac{P_{eq}}{F_a} = \frac{1432}{18} = 80\,\text{in.}^2$$

Note that Eq. (7.25), in general, overestimates the design requirement.

Try W14x233

$I_x = 3010\,\text{in.}^4$	$A = 68.5\,\text{in.}^2$	$b_f = 15.89\,\text{in.}$
$I_y = 1150\,\text{in.}^4$	$S_x = 375\,\text{in.}^3$	$r_x = 6.63\,\text{in.}$
$r_T = 4.40\,\text{in.}$	$S_y = 145\,\text{in.}^3$	$r_y = 4.10\,\text{in.}$
$\frac{d}{A_f} = 0.59\,1/\text{in.}$	$\frac{b_f}{2t_f} = 4.6$	$\frac{d}{t_w} = 15.0$

To find the effective length factors K_x and K_y we use the alignment chart [Fig. 7.4(b)].

For W21x83: $I_x = 1830\text{ in.}^4$
For W18x55: $I_x = 890\text{ in.}^4$

$$(G_A)_x = (G_B)_x = \frac{\sum I_c / L_c}{\sum I_g / L_g} = \frac{2(3010/12)}{2(1830/25)} = 3.4$$

$$(G_A)_y = (G_B)_y = \frac{2(1150/12)}{890/25} = 5.4$$

$G_{inelastic} = G_{elastic}(SRF)$ (page 3-6 ASD Manual)

SRF = Stiffness Reduction Factor

$$f_a = \frac{P}{A} = \frac{400}{68.5} = 5.84 \text{ ksi}$$

$\therefore$ No reduction of G's is necessary.

From the alignment chart [Fig. 7.4(b)]:

$K_x = 1.9$ $\qquad$ $K_y = 2.3$

$$\left(\frac{KL}{r}\right)_x = \frac{(1.9)(12)(12)}{6.63} = 41.3$$

$$\left(\frac{KL}{r}\right)_y = \frac{(2.3)(12)(12)}{4.10} = 80.8$$

(controls the value of allowable axial compressive stress, F_a)

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}} = \sqrt{\frac{2\pi^2 (29000)}{(36)}} = 126.1 > \left(\frac{KL}{r}\right)_y$$

$$\text{F.S.} = \frac{5}{3} + \frac{3(KL/r)_y}{8C_c} - \frac{(KL/r)_y^3}{8C_c^3}$$

$$= \frac{5}{3} + \frac{3(80.8)}{8(126.1)} - \frac{(80.8)^3}{8(126.1)^3} = 1.874$$

$$F_a = \frac{\left[1 - \frac{(KL/r)_y^2}{2C_c^2}\right]F_y}{\text{F.S.}} = \frac{\left[1 - \frac{1}{2}\left(\frac{80.8}{126.1}\right)^2\right](36)}{1.874} = 15.26 \text{ ksi}$$

$$f_a = \frac{P}{A} = \frac{400}{68.5} = 5.84 \text{ ksi}$$

$$\frac{f_a}{F_a} = \frac{5.84}{15.26} > 0.15$$

$\therefore$ **Equations (7.5) and (7.6) must be satisfied.**

Check whether $F_{bx} = 0.66F_y$ (ASD F1.1 and Table B5.1):

$$L_u = (12)(12) = 144 \text{ in.}$$

$$\frac{76b_f}{\sqrt{F_y}} = \frac{76(15.89)}{\sqrt{36}} = 201.3 \text{ in.} > L_u = 144 \text{ in.}$$ ASD F1.1

$$\frac{20{,}000}{(d / A_f)F_y} = \frac{20{,}000}{(0.59)(36)} = 941.6 > L_u = 144 \text{ in.}$$ ASD F1.1

$$\frac{b_f}{t_f} = 9.12 < \frac{130}{\sqrt{F_y}} = 21.7$$ ASD Table B5.1

$$\frac{f_a}{F_y} = \frac{5.84}{36} = 0.162 > 0.16$$

$$\frac{d}{t_w} = \frac{16.04}{1.07} = 15.0 < \frac{257}{\sqrt{F_y}} = 42.8$$ ASD Table B5.1

$$F_{bx} = 0.66F_y = 24 \text{ ksi}$$

$$F_{by} = 0.75F_y = 27 \text{ ksi}$$ ASD F2.1

$$F'_{ex} = \frac{12\pi^2 E}{23(KL/r)_x^2} = \frac{12\pi^2(29{,}000)}{23(41.3)^2} = 87.55 \text{ ksi}$$

$$F'_{ey} = \frac{12\pi^2 E}{23(KL/r)_y^2} = \frac{12\pi^2(29{,}000)}{23(80.8)^2} = 22.87 \text{ ksi}$$

$$C_{mx} = C_{my} = 0.85$$

$$f_{bx} = \frac{M_x}{S_x} = \frac{(200)(12)}{375} = 6.40 \text{ ksi}$$

$$f_{by} = \frac{M_y}{S_y} = \frac{(100)(12)}{145} = 8.28 \text{ ksi}$$

Check Eq. (7.5):

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}}$$

$$= \frac{5.84}{15.26} + \frac{0.85(6.4)}{\left(1 - \frac{5.84}{87.55}\right) 24} + \frac{0.85(8.28)}{\left(1 - \frac{5.84}{22.87}\right) 27} = 0.98 < 1$$

Check Eq. (7.6):

$$\frac{f_a}{0.60F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} = \frac{5.84}{22} + \frac{6.4}{24} + \frac{8.28}{27} = 0.84 \le 1.0$$

USE W14x233

Example 2

Design a 10x10 square tube from the ASD manual for a column with a length of 20 ft and subjected to an axial load of 400 Kips and a uniformly distributed load of intensity 0.2 Kips/ft acting on one of its sides (Fig. 7.6). The column is used in a braced frame. Assume that the column ends are pinned. Use steel with yield stress of 60 ksi.

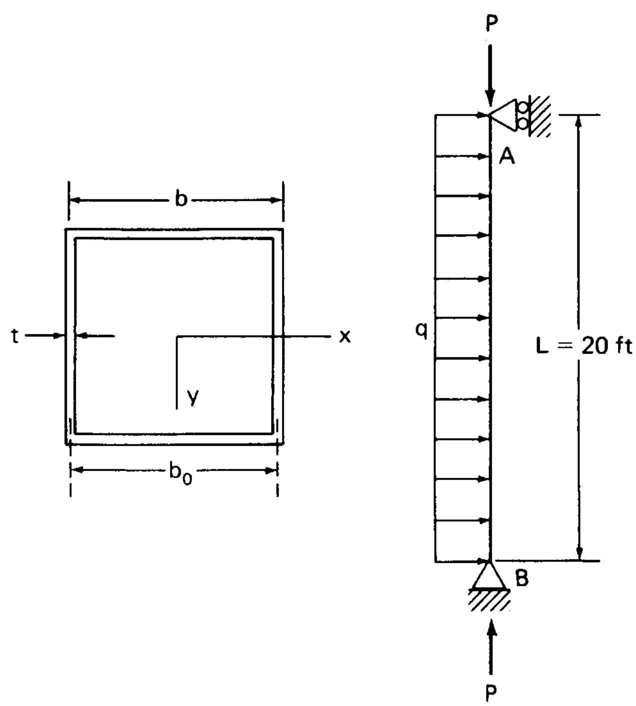


Figure 7.6

Solution

P = 400 Kips q = 0.2 Kips/ft K = 1 L = 20 ft

$$M_y = \frac{qL^2}{8} = \frac{0.2(20)^2}{8} = 10\ \text{K - ft} = 120\ \text{K - in}$$

The properties of a square box section can be found in terms of its thickness t and width of the section b_o (distance between the midplanes of the opposite plates) as follows:

$$A = 4b_o t$$

$$S_x = S_y = \frac{4b_o t(b_o^2 + t^2)}{3(b_o + t)}$$

$$r_x = r_y = \sqrt{\frac{b_o^2 + t^2}{6}}$$

$$B_x = B_y = \frac{3(b_o + t)}{b_o^2 + t^2}$$

For the first trial, considering that t is small compared with b, we can write

$$B_y \approx \frac{3}{b_o} \approx \frac{3}{b} = \frac{3}{10} = 0.3\ \text{1/in.}$$

$$r_x = r_y \approx \frac{b_o}{\sqrt{6}} \approx 0.4b = 0.4(10) = 4\ \text{in.}$$

Note, the equivalent axial load form Eq. (7.25) is equal to

$$P_{eq} = P + B_y M_y = 400 + 0.3(120) = 436 \text{ Kips}$$

$$\frac{KL}{r} = \frac{(20)(12)}{4} = 60$$

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}} = \sqrt{\frac{2\pi^2(29000)}{(60)}} = 97.7 > \frac{KL}{r}$$

Thus, the allowable axial compressive stress from Eq. (6.14) becomes F_a = 26.03 ksi.

$$\text{Required A} \approx \frac{P_{eq}}{F_a} = \frac{436}{26.03} = 16.75 \text{ in.}^2$$

Try TUBE 10x10 with wall thickness t = 0.5 in.

$$A = 18.4 \text{ in.}^2 \qquad S_x = S_y = 54.2 \text{ in.}^3 \qquad r_x = r_y = 3.84 \text{ in.}$$

Check for local flange buckling (ASD Table B5.1):

$$\frac{b_f}{t_f} = \frac{10}{0.5} = 20 < \frac{238}{\sqrt{F_y}} = \frac{238}{\sqrt{60}} = 30.7$$

From Table 7.1 (Case a): C_{my} = 1.0

$$\frac{KL}{r} = \frac{(20)(12)}{3.84} = 62.5 < C_c = 97.7$$

From Eq. (6.14): F_a = 25.52 ksi ASD E2

$$f_a = \frac{P}{A} = \frac{400}{18.4} = 21.74\,\text{ksi}$$

$$\frac{f_a}{F_a} = \frac{21.74}{25.52} = 0.852 > 0.15$$

$$f_{by} = \frac{M_y}{S_y} = \frac{120}{54.2} = 2.21\,\text{ksi}$$

$$F'_{ey} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29{,}000)}{23(62.5)^2} = 38.25\,\text{ksi}$$

Check whether $F_b = 0.66F_y$ (ASD F3.1 and Table B5.1):

$$\frac{d}{b} = 1 < 6$$

$$\frac{t_f}{t_w} = 1 < 2$$

$$L_u = 240\ \text{in.} < \frac{1950b}{F_y} = \frac{1950(10)}{60} = 325\,\text{in.}$$

$$\frac{b_f}{t_f} = 20 < \frac{190}{\sqrt{F_y}} = \frac{190}{\sqrt{60}} = 24.5$$

$$\frac{f_a}{F_y} = \frac{21.74}{60} = 0.36 > 0.16$$

$$\frac{d}{t_w} = \frac{10}{0.5} = 20 < \frac{257}{\sqrt{F_y}} = \frac{257}{\sqrt{60}} = 33.2$$

$\therefore\ F_{by} = 0.66F_y = 0.66(60) = 40$ ksi

Check Eq. (7.5):

$$\frac{f_a}{F_a} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}} = 0.852 + \frac{(1.0)(2.21)}{\left(1 - \frac{21.74}{38.23}\right)(40)} = 0.98 < 1$$

USE TUBE 10x10, t = 0.5 in.

7.7 LOAD AND RESISTANCE FACTOR DESIGN OF BEAM-COLUMNS

7.7.1 METHOD ONE FOR MEMBERS IN BRACED AND UNBRACED FRAMES

According to LRFD H1.1 and C1, members subjected to combined axial force and bending shall be proportioned to satisfy the following interaction equations:

$$\text{For } \frac{P_u}{\phi_c P_n} \geq 0.2: \quad \frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \tag{7.29}$$

$$\text{For } \frac{P_u}{\phi_c P_n} < 0.2: \quad \frac{P_u}{2\phi_c P_n} + \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \leq 1.0 \tag{7.30}$$

$$M_u = B_1 M_{NT} + B_2 M_{LT} \tag{7.31}$$

where

P_n = nominal axial load strength
P_u = required axial load strength
M_n = nominal flexural strength determined according to section 5.10
M_u = required flexural strength
M_{NT} = required flexural strength of member assuming there is no lateral translation
M_{LT} = required flexural strength of member as a result of lateral translation of the frame only
B_1 = the member instability amplification factor
B_2 = the frame instability amplification factor

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} \geq 1.0 \tag{7.32}$$

$$B_2 = \frac{1}{1 - \frac{\sum P_u \Delta_{OH}}{\sum H L}} \tag{7.33}$$

or

$$B_2 = \frac{1}{1 - \frac{\sum P_u}{\sum P_{e2}}} \tag{7.34}$$

$\sum P_u$ = Required axial load strength of all columns in a story

$$P_{e1} = \frac{AF_y}{\lambda_{c1}^2} \tag{7.35}$$

$\lambda_{c1} = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}}$ is the slenderness parameter when the effective length factor K in the plane of bending is found for the braced frame

Δ_{OH} = lateral inter-story displacement of the story under consideration
$\sum H$ = sum of the story horizontal forces producing Δ_{OH}
L = story height

$$P_{e2} = \frac{AF_y}{\lambda_{c2}^2} \tag{7.36}$$

$\lambda_{c2} = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}}$ is the slenderness parameter when the effective length factor K in the plane of bending is found for the unbraced frame

The axial force times the deflection produced by primary moments (due to transverse loads or end moments acting on the member) causes additional moments referred to as secondary or P-δ moment. The B_1 factor in Eq. (7.31) takes care of this secondary moment.

In unbraced frames, vertical gravity loads times the drift (lateral displacement) produces overturning moment and additional drift. The B_2 factor in Eq. (7.31) takes care of this instability effect, referred to as PΔ effect. The PΔ effect in braced frame is negligible (B_2 = 0).

The coefficient C_m for members not subjected to transverse loading between their ends is found by the same Eq. (7.8) provided by the ASD code. Where there is transverse loading on the member, however, the LRFD code makes the following simplification in lieu of a “rational analysis”:

For members with simple ends: C_m = 1.0
For members with end restraints: C_m = 0.85

7.7.2 METHOD TWO FOR MEMBERS IN BRACED FRAMES

For I shapes with $b_f / d \leq 1.0$ in braced frames subjected to combined axial force and biaxial bending, LRFD Appendix H3 provides alternate nonlinear interaction equations as follows:

$$\left(\frac{M_{ux}}{\phi_b M'_{px}}\right)^{\xi} + \left(\frac{M_{uy}}{\phi_b M'_{py}}\right)^{\xi} \leq 1.0 \tag{7.37}$$

$$\left(\frac{C_{mx} M_{ux}}{\phi_b M'_{nx}}\right)^{\eta} + \left(\frac{C_{my} M_{uy}}{\phi_b M'_{ny}}\right)^{\eta} \leq 1.0 \tag{7.38}$$

where

$$\xi = \begin{cases} 1.0 & \text{for } b_f / d < 0.5 \\ 1.6 - \dfrac{P_u / P_y}{2[\ln(P_u / P_y)]} & \text{for } 0.5 \le b_f / d \le 1.0 \end{cases} \tag{7.39}$$

$$\eta = \begin{cases} 1 & \text{for } b_f / d < 0.3 \\ 0.4 + \dfrac{P_u}{P_y} + \dfrac{b_f}{d} \ge 1.0 & \text{for } 0.3 \le b_f / d \le 1.0 \end{cases} \tag{7.40}$$

$$M'_{px} = 1.2 M_{px} [1 - (P_u / P_y)] \le M_{px} \tag{7.41}$$

$$M'_{py} = 1.2 M_{py} [1 - (P_u / P_y)^2] \le M_{py} \tag{7.42}$$

$$M'_{nx} = M_{nx} \left[1 - \left(\frac{P_u}{\phi_c P_n} \right) \right] [1 - (P_u / P_{ex})] \tag{7.43}$$

$$M'_{ny} = M_{ny} \left[1 - \left(\frac{P_u}{\phi_c P_n} \right) \right] [1 - (P_u / P_{ey})] \tag{7.44}$$

$$P_y = A F_y \tag{7.45}$$

M_{px} = plastic moment about the x - axis

M_{py} = plastic moment about the y - axis

Chen and Lui (1985) have compared the linear and nonlinear interaction equations. Except when b_f /d is small and the ratio of moment to axial force is large, they concluded, the nonlinear interaction equations yield more economical sections than the linear interaction equations.

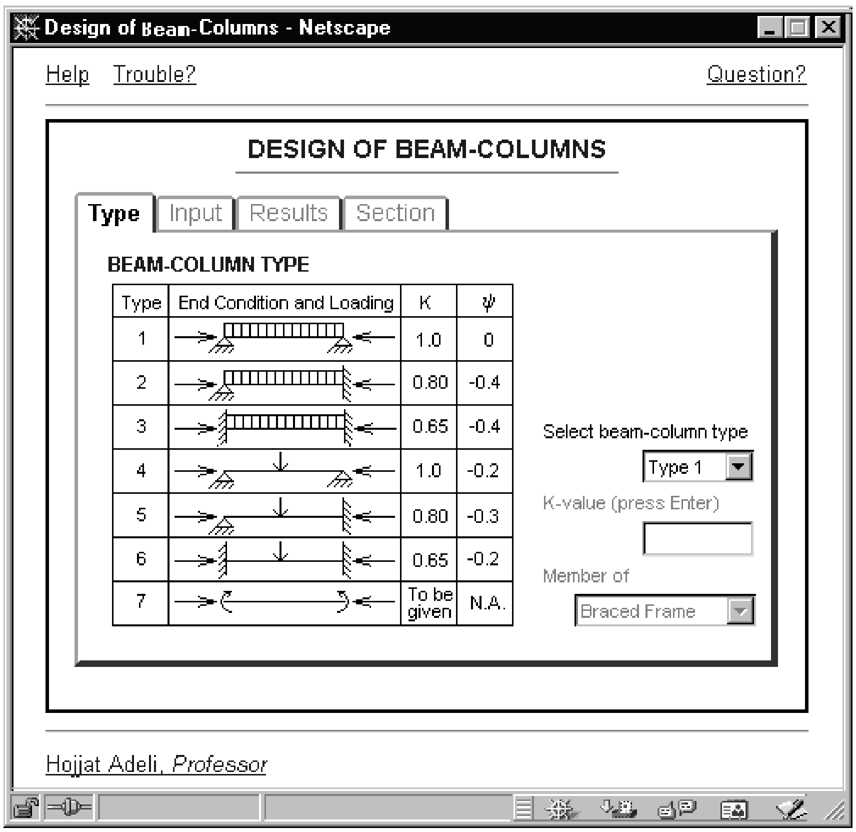


Figure 7.7 Beam-column type panel for the applet for design of beam-columns

7.8 WEB-BASED INTERACTIVE DESIGN OF BEAM-COLUMNS

The applet presented in this section is for interactive design of beam-columns with various end conditions according to the ASD and LRFD codes. This applet consists of four panels: *Beam-Column Type* (Fig. 7.7), *Input* (Fig. 7.8), *Results* (Fig. 7.9), and *Section* (Fig. 7.10). The

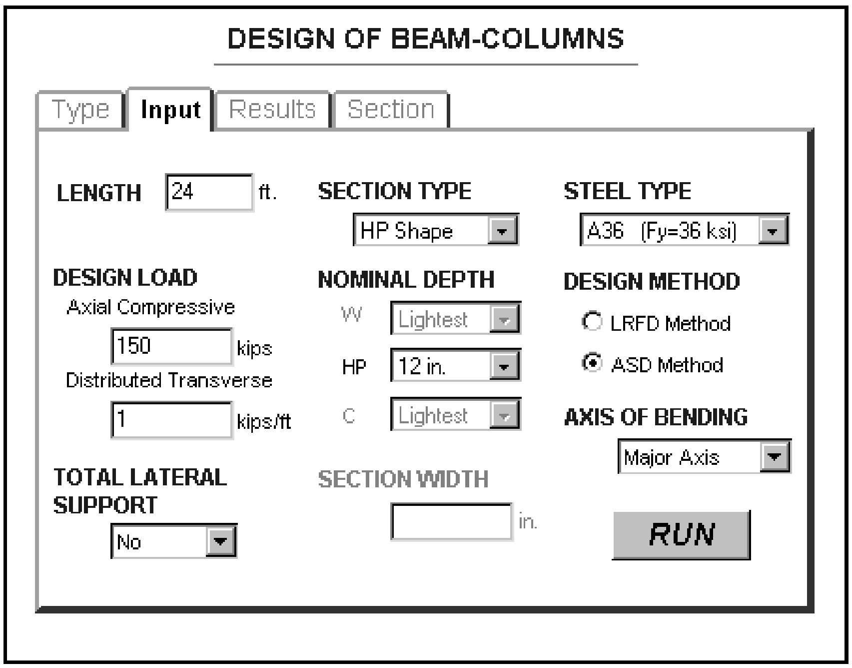


Figure 7.8 Input panel

user can select one of six different cases of beam-column types presented in Table 7.1 plus one more case where the user can choose the end moments (Fig. 7.11).

For type 7 beam-column, the user needs to enter the design K-value and to specify whether the beam-column is a member of a braced frame or an unbraced frame; i.e. frame braced against sidesway or subjected to sidesway (Fig 7.12). Those two items are not active unless the user selects type 7. Likewise, The user cannot change the design K-value for other types of beam-columns.

Depending on whether the sidesway is inhibited through bracing or not, the applet checks the design K-value given by the user and

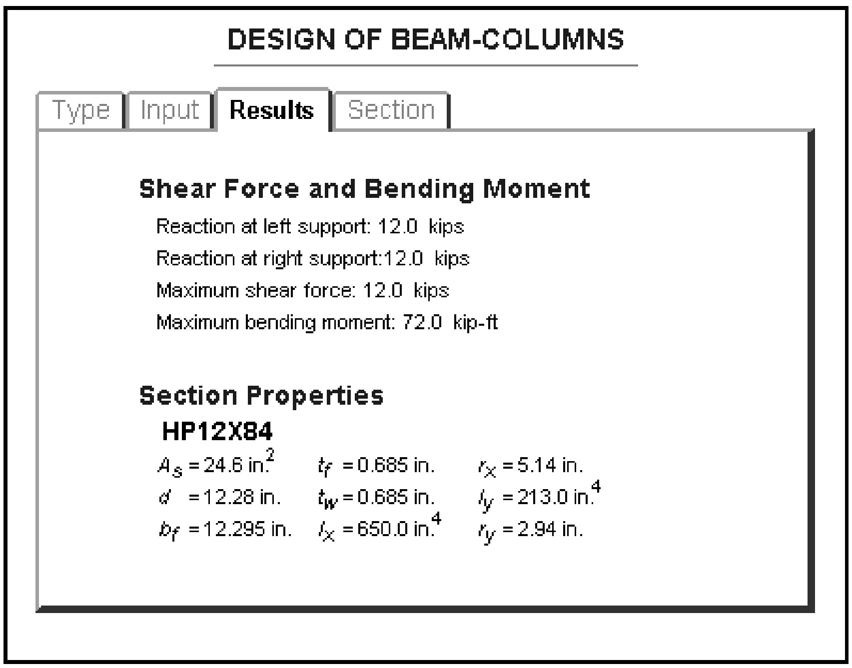


Figure 7.9 Results panel

presents a warning message when the user enters a value out of range (Fig. 7.13 and Fig 7.14). A similar warning message will be presented when the user tries to run the applet without specifying the design K-value for type 7 beam-column.

The applet displays different input menus according to the beam-column type selected by the user. For types 1 to 3, the design load consists of an axial load and a distributed transverse load (Figs. 7.8 and 7.15). If the user selects the LRFD method as a design method, the applet asks the user to enter the factored design load (Fig. 7.15). An axial load and a concentrated transverse load at the mid-span comprise the

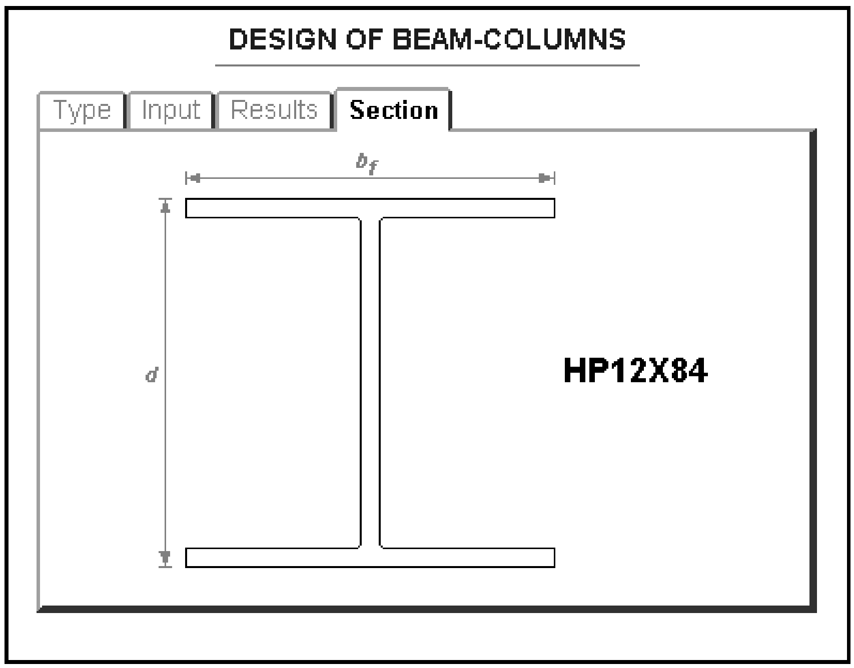


Figure 7.10 Section panel

design load for types 4 to 6, and an axial load and moments at left and right ends for type 7 as shown in Figs. 7.16, and 7.17, respectively. The user can specify clockwise (CW) or counter clockwise (CCW) moment by clicking on the choice list (Fig. 7.17).

The user can select one of six different types of cross sections (see Fig. 6.14) and may or may not specify the nominal depth of cross section, similar to the applet for design of axially loaded compression members described in section 6.7. At this time, the applet has the following limitations:

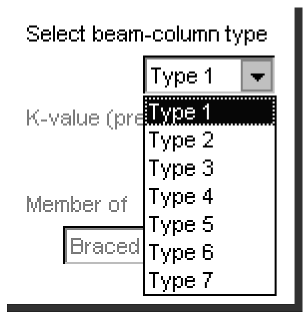


Figure 7.11

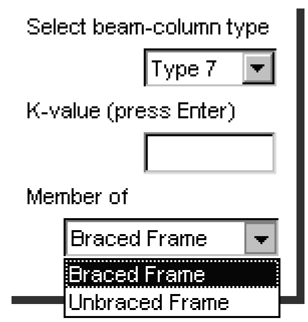


Figure 7.12

1. The member can be subjected to bending about major or minor axis only.

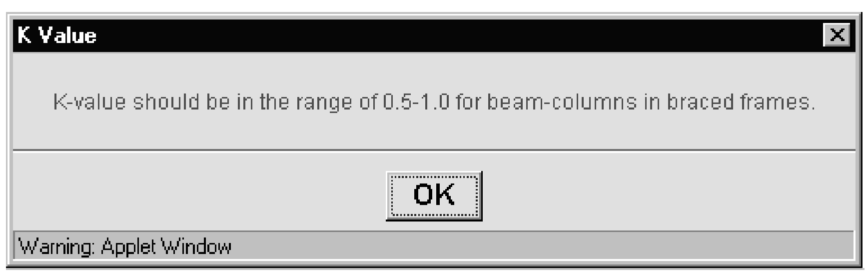


Figure 7.13

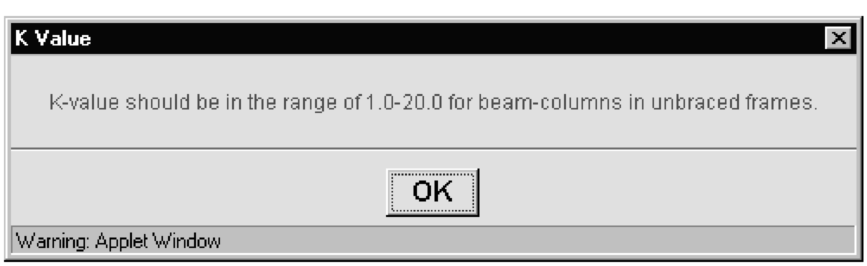


Figure 7.14

2. The member can either have full lateral support or no lateral support at all. Intermediate supports are not allowed.
3. The effective length factor is the same for bending in both principal planes.
4. The end conditions and loads acting on the member are limited to those cases shown in the beam-column type panel shown in Fig. 7.7.

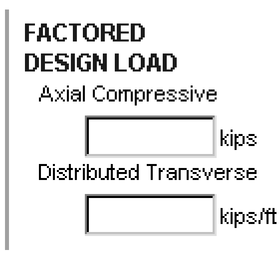


Figure 7.15

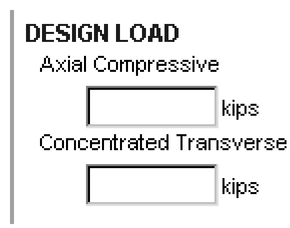


Figure 7.16

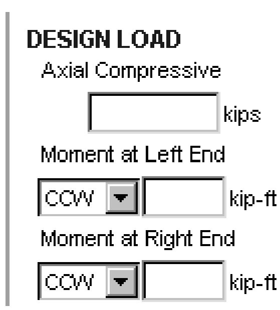


Figure 7.17

7.9 PROBLEMS

7.1 Design a W14 section made of A36 steel with yield stress of 36 ksi for the beam-column shown in Fig. 7.16. The member is subjected to an axial compressive load of 150 Kips. Bending about the major axis is due to a uniformly distributed load of intensity q = 1 K/ft. Bending about the minor axis is due to a couple of magnitude 12 K-ft applied at the midpoint C of the member. Assume lateral support at the two ends A and B only.

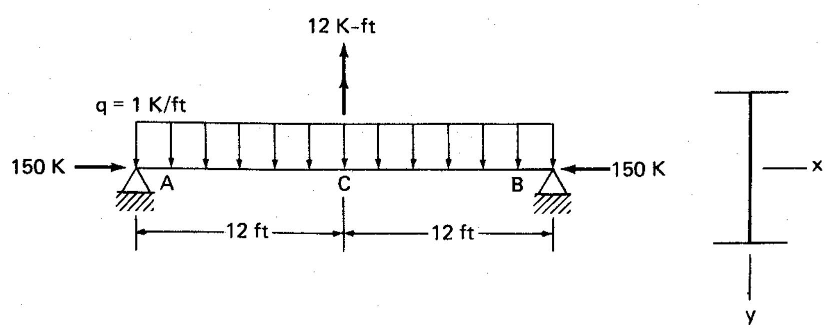


Figure 7.16

7.2 The W-shape column shown in Fig. 7.17 is part of a laterally braced frame. It is subjected to two centric axial compressive forces of P_1 and P_3 at the two ends and the force P_2 with an eccentricity of 12 in. with respect to the center of the column. The three forces are applied in the plane of the web. In addition to the two hinged ends A and B, the column also has lateral support in the x-direction at point D at a distance of 8 ft from the top end A. The column is made of steel with yield stress of 50 ksi. Check the adequacy of a W14x43 section for the column. Neglect the weight of the column.

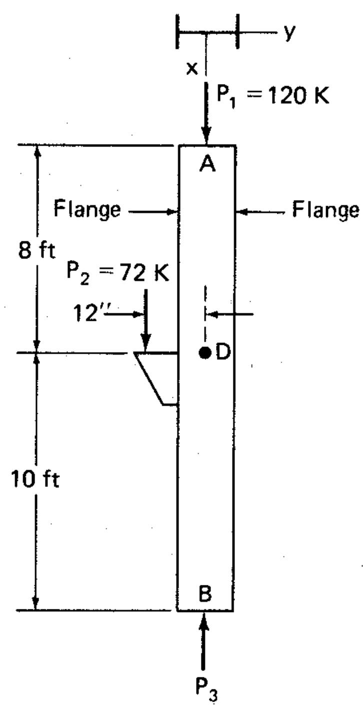


Figure 7.17

7.3 Design the moment-resisting frame shown in Fig. 7.18 using A36 steel with yield stress of 36 ksi. Lateral support is provided at points *B*, *C*, *E* (midpoint of column *AB*), *F* (midpoint of column *CD*), and *G* (midpoint of beam *BC*). Select the lightest W14 for the column and the lightest W section for the beam.

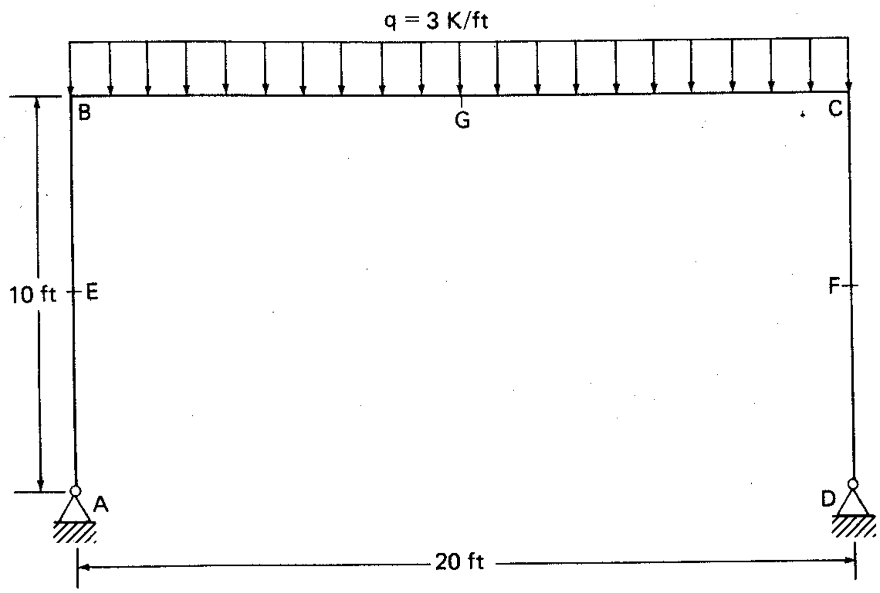


Figure 7.18

7.4 Find the lightest W12 section for the column in Example 1 of this chapter.

7.5 Design a 12x12 tube for Example 2 of this chapter.

7.6 Solve Problem 5.17 as a beam-column subjected to axial compressive forces of 20 K at the two ends in addition to the lateral vertical and horizontal loads.

7.7 Solve Problem 5.18 as a beam-column subjected to axial compressive forces of 25 K at the two ends in addition to the lateral vertical and horizontal loads.