5 Design of Beams

5.1 TYPES OF BEAMS

5.1.1 Standard Hot-Rolled Sections [Fig. 5.1(a)]

Standard hot-rolled sections in general and wide-flange (or W) sections in particular are the most popular types of beam sections. For a given weight, W sections have larger section moduli than S sections and consequently in general are more economical. S sections may be used in the following situations:

1. For crane rails where larger flange thickness near the web may be advantageous for lateral bending.
2. For clearance or other reason, narrow flanges may be required.
3. Where shear forces are considerably high.

A W shape bent about its minor axis may be as little as 5 percent as strong as the same shape bent about its major axis.

Channels are used in the following situations:

1. Where the load on the beam is light.
2. Where narrow flanges are desirable for clearance or other reasons.

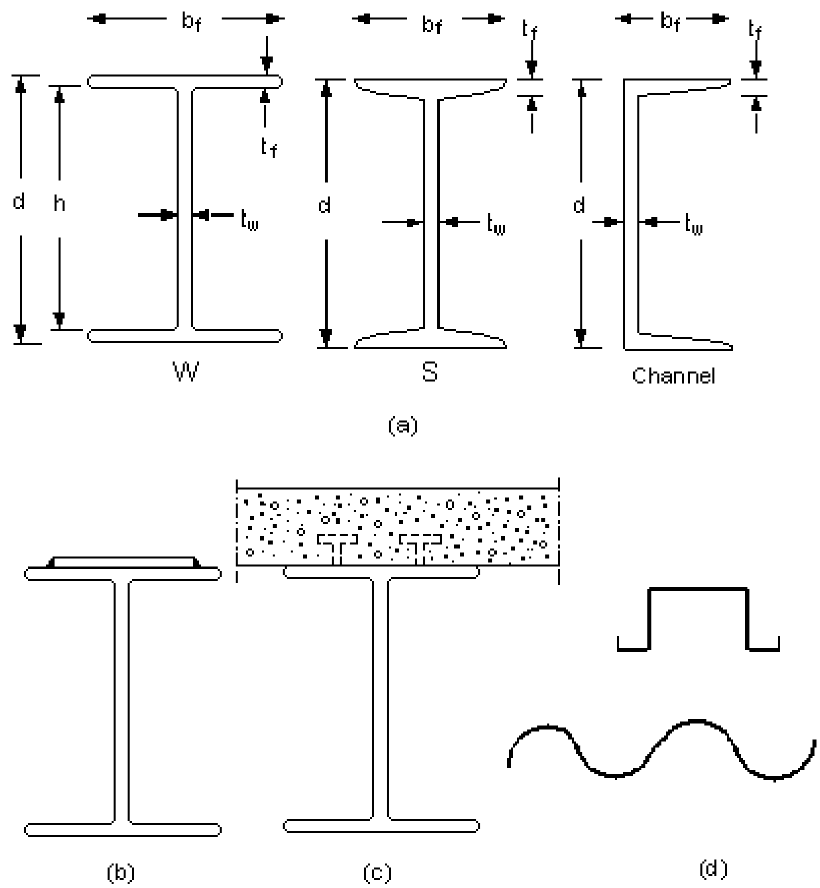


Figure 5.1 Different type of beams. a) standard hot-rolled sections, b) plate covered section, c) composite beam, d) sheet beams.

Channels should be avoided where lateral loading is present.

Note that in Fig. 1(a), d is the depth of the cross section, h and t_w are depth and thickness of the web, respectively, and b_f and t_f are the width and the thickness of the flange, respectively.

5.1.2 Plate-Covered Section [Fig. 5.1(b)]

The bending capacity of available rolled sections may be increased by adding plate(s) to the section.

5.1.3 Plate and Box girders [Figs 9.2 and 9.5]

For girders covering large spans and/or carrying heavy loads, the available standard rolled sections may be inadequate. In this case, the designer may use a plate or box girder made of steel plates. These girders may be homogeneous, made of a single grade of steel, or hybrid, made of high-strength flanges. Design of plate girders is covered in detail in Chapter 9.

5.1.4 Composite Beams [Fig. 5.1(c)]

If the steel beam is covered with a concrete deck, the designer may take advantage of the additional strength of the concrete, provided that the concrete deck is properly fastened to the steel beam (for example, by shear studs).

5.1.5 Sheet Beams [Fig. 5.1(d)]

These beams are made of thin cold-formed steel sheets in a variety of shapes. They are useful for very light loads (for example, light roofs).

5.2 REVIEW OF BEAM THEORY

5.2.1 Introduction

In this section, and in fact throughout this volume, we assume that the cross section of the beam has at least one axis of symmetry. The plane containing the axis of symmetry and the longitudinal axis of the beam is called the *plane of symmetry*. Beams are subjected to loads with resultants acting in their plane of symmetry and transverse to their longitudinal axis. Thus, beams carry the loads basically through bending.

In the design of beams, the following considerations are necessary:

1. Bending stresses
2. Shearing stresses
3. Local buckling
4. Lateral torsional buckling
5. Web crippling
6. Deflection

Initial selection of the beam is usually made on the basis of the maximum bending stresses. The other design requirements are checked subsequently.

5.2.2 Elastic Bending of Beams

In elastic theory of bending, the following assumptions are usually made:

1. Deflections are small and change of geometry is negligible.
2. Plane sections remain plane after bending.
3. Shear deformations are small
4. Interaction of axial forces and bending is negligible.
5. Buckling and stability of the beam is not a problem.
6. The material is linearly elastic; that is, the stress is proportional to the strain.

Assumption 2 implies that the variation of the strain ε over the cross section is linear [Fig. 2(b)], and we can write:

$$\varepsilon = Ky = \frac{y}{\rho} \tag{5.1}$$

where ρ is the radius of curvature and K is the curvature given in terms of the bending moment at the section M, the modulus of elasticity E, and the moment of inertia of the cross section about the axis of bending I_x,

$$K = \frac{1}{\rho} = \frac{M}{EI_x} \tag{5.2}$$

Based on assumption 6 and using Eqs. (5.1) and (5.2), the normal stress due to bending is

$$f_b = E\varepsilon = EKy = \frac{My}{I_x} \tag{5.3}$$

Denoting the maximum value of the bending moment over the beam length by $M_{\max}$ and the maximum y distance from the elastic

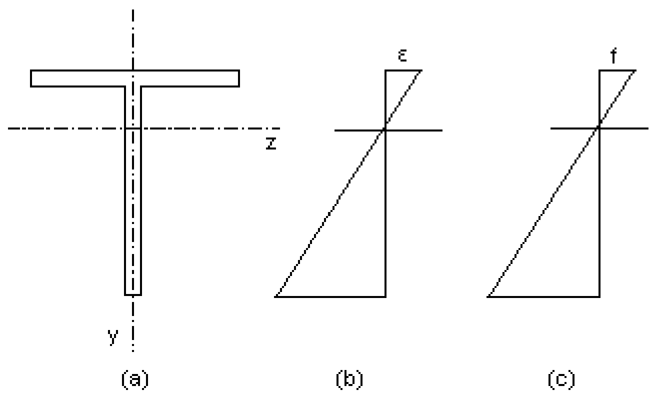


Figure 5.2 Strain ε and elastic stress *f* distribution over a cross section of a beam.

neutral axis (E.N.A.) or centroidal axis by *c*, the maximum bending stress will be

$$(f_b)_{\max} = \frac{M_{\max} c}{I_x} = \frac{M_{\max}}{S_x} \tag{5.4}$$

where S_x is called the minimum section modulus of the section in the case of beams with one axis of symmetry and the section modulus for the case of doubly symmetric beams. If the allowable bending stress as specified by the ASD code is F_b, the required section modulus will be

$$S_x = \frac{M_{\max}}{F_b} \tag{5.5}$$

5.2.3 Shear Stresses

Shear stresses are usually not a controlling factor in the design of beams, except for the following cases:

1. The beam is very short.
2. There are holes in the web of the beam. These holes may be for passing electrical and mechanical ducts or for increasing the bending strength in case of castellated beams (Fig. 5.3).
3. The beam is subjected to a very heavy concentrated load near one of the supports.
4. The beam is coped, as shown in Fig. 5.4.

Generally speaking, the flanges of the beam carry the normal stresses due to bending moments, and the web carries the shear stresses due to shear force. The distribution of shear stress over the depth of an I-

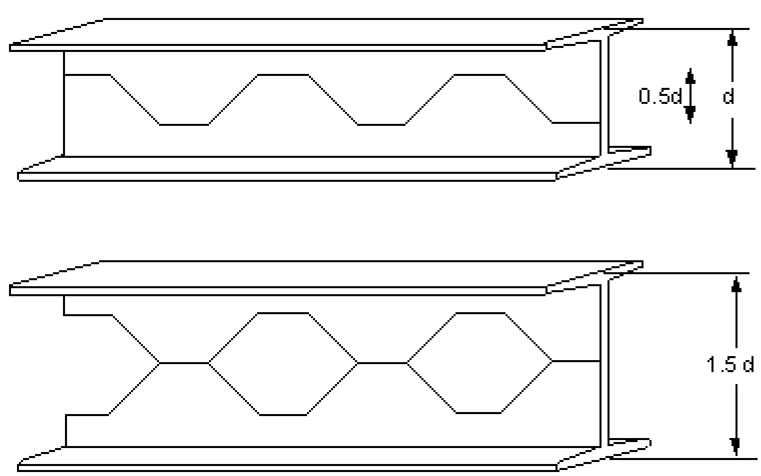


Figure 5.3 Castellated beam

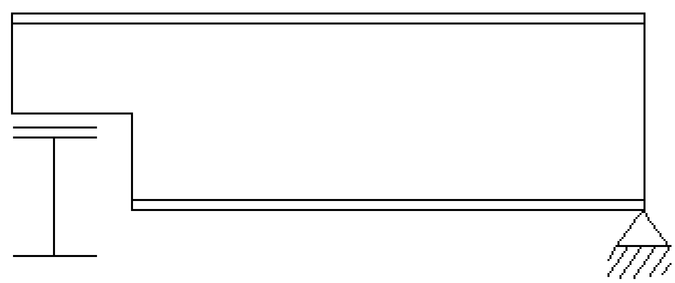


Figure 5.4 Beam with a coped end

section is shown in Fig. 5.5. The shear stress at any point in the web is found from

$$f_v = \frac{VQ}{I_x t} \tag{5.6}$$

where

V = transverse shear on the cross section

Q = first moment of area of the cross section above (or below) the point at which the shear stress is computed, about the neutral axis

t = thickness of the web where the shear stress is calculated

Considering that the variation of the shear over the depth of the web is small, it is a common practice to use the following simple equation for computing the shear stress in the web:

$$f_v = \frac{V}{ht} \tag{5.7}$$

where h is the depth of the web, given in terms of the total depth of the beam, d, and the thickness of the flange, t_f.

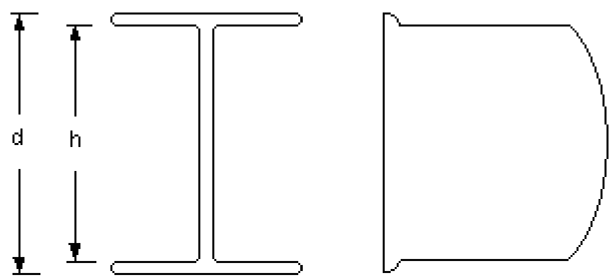


Figure 5.5 Shear stress distribution over the cross section of an I-beam

$$h = d - 2t_f \tag{5.8}$$

The maximum shear stress over the span of the beam should be less than the allowable shear stress F_v .

5.3 LOCAL BUCKLING

The hot-rolled steel sections are thin-walled sections consisting of a number of thin plates. When normal stresses due to bending and/or direct axial forces are large, each plate (for example, flange plate or web plate) may buckle locally in a plane perpendicular to its plane.

In order to prevent this undesirable phenomenon, the width-to-thickness ratios of the thin flange and the web plates are limited by the code. These limitations according to the ASD code, expressed in terms of the yield stress, F_y, are summarized in Table 5.1. The hot-rolled thin-walled sections are divided into three categories:

TABLE 5.1 LOCAL BUCKLING WIDTH-THICKNESS LIMITATIONS ACCORDING TO THE ASD CODE (ASD TABLE B5.1)

(1) Types of element	(2) Width-thickness ratio	(3) Compact	(4) Noncompact
Flanges of I-section, tees, and channels	$\frac{b_f}{t_f}$	$\frac{130}{\sqrt{F_y}}$ [c]	$\frac{190}{\sqrt{F_y}}$ [c]
Webs of I-sections, channels, and hollow rectangular sections in flexure	$\frac{h}{t_w}$	$\frac{640}{\sqrt{F_y}}$ [a]	$\frac{760}{\sqrt{F_b}}$ [b]
Stems of tees	$\frac{d}{t_w}$	NA	$\frac{127}{\sqrt{F_y}}$
Flanges of hollow rectangular sections	$\frac{b_f}{t_f}$	$\frac{190}{\sqrt{F_y}}$	$\frac{238}{\sqrt{F_y}}$
Single-angle struts, double-angle struts with separators	$\frac{b}{t}$	NA	$\frac{76}{\sqrt{F_y}}$
Hollow circular sections	$\frac{D}{t}$	$\frac{3300}{F_y}$	—

[a] When, in addition to bending, the section is also acted on by an axial compressive stress f_a, the limiting width-thickness ratio becomes:

$$\frac{h}{t_w} = \begin{cases} \frac{640}{\sqrt{F_y}}\left(1-3.74\frac{f_a}{F_y}\right) & \text{when } \frac{f_a}{F_y} \le 0.16 \\ \frac{257}{\sqrt{F_y}} & \text{when } \frac{f_a}{F_y} > 0.16 \end{cases}$$

[b] For uniformly compressed webs of columns: $\frac{h}{t_w} = \frac{253}{\sqrt{F_y}}$

[c] For channels divide by 2.

1. Compact sections
2. Noncompact sections
3. Sections with slender compression elements

Compact sections are expected to develop their full plastic moment capacity, M_p. The width-thickness ratios for these sections are in the third column of Table 5.1. When the width-thickness ratios for a section exceed these limits, the section is called *noncompact*. The limiting ratios for the noncompact sections are given in column 4 of Table 5.1. When the width-thickness ratios exceed these values, the section is referred to as a section with slender compression elements.

5.4 LATERAL TORSIONAL BUCKLING

The compression flange of a beam behaves like an axially loaded column. Thus, in beams covering long spans the compression flange may tend to buckle. This tendency, however, is resisted by the tension flange to a certain extent. The overall effect is a phenomenon known as *lateral torsional buckling*, in which the beam tends to twist and displace laterally. Lateral torsional buckling may be prevented through the following provisions:

1. Lateral supports at intermediate points in addition to lateral supports at the vertical supports
2. Using torsionally strong sections (for example, box sections)
3. I-sections with relatively wide flanges

There are a number of methods for providing lateral supports. If a concrete slab is used, the steel beam may be embedded in the concrete slab as shown in Fig. 5.6. Simply placing the concrete slab on the beam does not provide sufficient lateral support. In this case, the beam should be anchored to the concrete slab by studs as shown in Fig. 5.7. When light floor decks are used, lateral bracing may be provided by cross-bracing in the plane of the floor, as shown in Fig. 5.8.

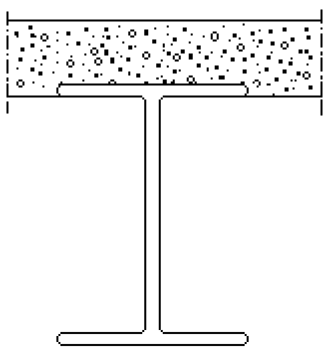


Figure 5.6 Steel beam embedded in the concrete slab

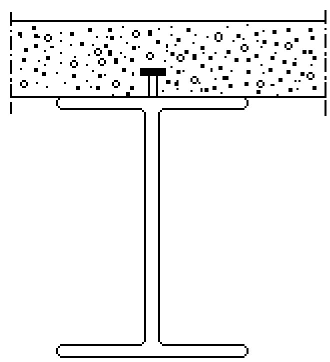


Figure 5.7 Steel beam connected to the concrete slab by studs

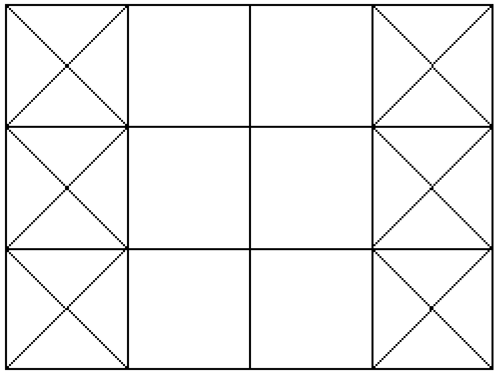


Figure 5.8 Cross-bracing in the plane of the floor

When sufficient lateral support is not provided, the phenomenon of lateral torsional buckling is prevented in actual design of steel structures by reducing the allowable bending stresses. In other words, the allowable bending stress is specified as a function of the unbraced length of the beam L_u, as discussed in the following section. The larger the unbraced length, the lower the allowable bending stress, and vice versa.

5.5 ALLOWABLE BENDING STRESSES ACCORDING TO THE ASD CODE

5.5.1 Bending About the Major Axis

5.5.1.1 Compact sections

For a member to qualify as a compact section, it must satisfy the following requirements (ASD F1.1 and F3.1):

1. The beam shall not be hybrid.
2. The beam shall not be made of steel with a yield stress of greater than 65 ksi.
3. The flanges shall be continuously connected to the web(s).
4. The width-thickness ratio of the compression flange and the web shall not exceed the limiting values given in the third column of Table 5.1.
5. The laterally unsupported length L_u of the compression flange shall satisfy the following requirements:

   a. For I and T sections and channels

$$L_u \le \frac{76 b_f}{\sqrt{F_y}} \tag{5.9}$$

$$L_u \le \frac{20{,}000}{(d / A_f) F_y} \tag{5.10}$$

b. For rectangular box members

$$\frac{d}{b} \le 6 \tag{5.11}$$

$$\frac{t_f}{t_w} \le 2 \tag{5.12}$$

$$L_u \le \left(1950 + 1200 \frac{M_1}{M_2} \right) \frac{b}{F_y} \tag{5.13}$$

where d is the depth of the cross section, b is its width, M_1 is the smaller and M_2 is the larger bending moment at the ends of the unbraced length with respect to the strong axis of bending, the ratio M_1/M_2 is positive when M_1 and M_2 have the same sign (double curvature bending) and negative otherwise (single curvature bending). The unbraced length L_u need not be less than $1200b/ F_y$.

The allowable bending stress for a compact section is given by:

$$F_b = 0.66 F_y \tag{5.14}$$

Note that when local flange and web buckling is not a problem and sufficient lateral support is provided to prevent lateral torsional buckling, the ‘basic’ allowable bending stress is $0.60F_y$. For compact sections, however, this value is increased by 10 percent, because compact sections can develop their full plastic moment capacity.

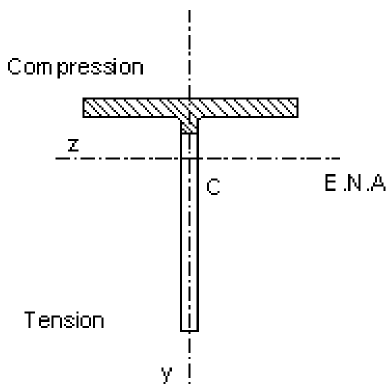


Figure 5.9 Portion of the cross section (shaded) for calculating r_T

5.5.1.2 Noncompact sections

For sections with width-thickness ratios not exceeding the limiting values given in the fourth column of Table 5.1, the allowable bending stress is the larger value computed from Eqs. (5.15) and (5.16), but not exceeding $0.60F_y$ (ASD F1.3)

$$F_b^{'} = \begin{cases} 0.60F_y & \dfrac{L_u}{r_T} < \sqrt{\dfrac{102{,}000C_b}{F_y}} \\ \left[\dfrac{2}{3} - \dfrac{F_y(L_u/r_T)^2}{1{,}530{,}000C_b}\right]F_y & \sqrt{\dfrac{102{,}000C_b}{F_y}} \le \dfrac{L_u}{r_T} < \sqrt{\dfrac{510{,}000C_b}{F_y}} \\ \dfrac{170{,}000C_b}{(L_u/r_T)^2} & \dfrac{L_u}{r_T} \ge \sqrt{\dfrac{510{,}000C_b}{F_y}} \end{cases} \tag{5.15}$$

$$F_b^{''} = \frac{12{,}000C_b}{L_u d/A_f} \le 0.60F_y \tag{5.16}$$

where

r_T = radius of gyration of the portion of the cross section consisting of the compression flange plus one-third of the compression web area (shaded area in Fig 5.9) about the axis in the web plane (y-axis in Fig 5.9)

A_f = area of the compression flange

C_b = $1.75 + 1.05(M_1 / M_2) + 0.3(M_1 / M_2)^2 \leq 2.3$ (5.17)

The ratio M_1/M_2 has been defined in the previous section. When the bending moment at a section within an unbraced length is greater than the two end moments, C_b shall be taken as unity.

Note 1. For channels, only Eq. (5.16) applies.

Note 2. When the area of the compression flange is less than that of the tension flange, only Eq. (5.15) applies.

Note 3. C_b can be conservatively assumed equal to one.

Note 4. For cantilever beam, C_b is assumed equal to one.

Note 5. For sections satisfying all the requirements of the compact section, except the width-thickness ratio for the flange, but satisfying the width-thickness limitation for noncompact sections given in the fourth column of Table 5.1, the allowable bending stress may be calculated from (ASD F1.2):

$$F_b = [0.79 - 0.002(b_f / 2t_f)\sqrt{F_y}]F_y$$

Note 6. Lateral torsional buckling for box sections with *d/b* less than or equal to 6 is not a problem. The allowable bending stress for these sections is F_b=0.60F_y, provided that the limiting width-thickness limitation for noncompact sections given in the fourth column of Table 5.1 are satisfied.

5.5.1.3 Sections with slender compression elements

In this section, we cover only the requirements for I-sections, channels, and tees. For angles and hollow rectangular section, the reader may refer to the ASD Appendix B.

When the width-thickness ratio of the compression elements of a beam exceeds the values given in the last column of Table 5.1, a reduction factor Q_s, is used in order to prevent local buckling (ASD Appendix B). In this case, the maximum allowable bending stress shall not exceed the applicable value given in section 5.5.1.2 nor the following value:

$$F_b = 0.6 F_y Q_s \tag{5.19}$$

where Q_s is given by the following expressions:

For compression flange of I-beam:

$$Q_s = \begin{cases} 1.293 - 0.00309(b_f / 2t_f)\sqrt{F_y / k_c} & \dfrac{190}{\sqrt{\dfrac{F_y}{k_c}}} < \dfrac{b_f}{t_f} < \dfrac{390}{\sqrt{\dfrac{F_y}{k_c}}} \\[3em] \dfrac{26{,}200 k_c}{F_y (b_f / 2t_f)^2} & \dfrac{b_f}{t_f} \geq \dfrac{390}{\sqrt{\dfrac{F_y}{k_c}}} \end{cases} \tag{5.20}$$

where

$$k_c = \frac{4.05}{(h/t)^{0.46}} \quad \text{if } \frac{h}{t} > 70, \text{ otherwise } k_c = 1.0$$

TABLE 5.2 LIMITING PROPORTIONS FOR CHANNELS AND TEES WITH SLENDER ELEMENTS

Section	$\frac{b_f}{d}$	$\frac{t_f}{t_w}$
Rolled channel	≤ 0.50	≤ 2.0
Built-up channel	≤ 0.25	≤ 3.0
Rolled tee	≥ 0.50	≥ 1.10
Built-up tee	≥ 0.50	≥ 1.25

For channels, $b_f/2t_f$ is replaced with b_f/t_f in Eq. (5.20).
For stems of tees:

$$Q_s = \begin{cases} 1.908 - 0.00715(d/t_w)\sqrt{F_y} & \dfrac{127}{\sqrt{F_y}} < \dfrac{d}{t_w} < \dfrac{176}{\sqrt{F_y}} \\ \dfrac{20{,}000}{F_y(d/t_w)^2} & \dfrac{d}{t_w} \geq \dfrac{176}{\sqrt{F_y}} \end{cases} \tag{5.21}$$

Channels and tees whose flange width-thickness ratio exceeds the value given in the fourth column of Table 5.1 should conform to the additional limits summarized in Table 5.2.

5.5.2 Bending about the Minor Axis

5.5.2.1 Compact sections (ASD F2 & F3)

For I sections (ASD F2.1):

$$F_b = 0.75F_y \tag{5.22}$$

For rectangular box and tubular sections (ASD 3.1):

$$F_b = 0.66F_y \tag{5.23}$$

5.5.2.2 Noncompact sections

For I sections and rectangular tubular sections, in general,

$$F_b = 0.60F_y \tag{5.24}$$

For I sections satisfying all the requirements of the compact sections except the width-thickness ratio for the flange, but satisfying the width-thickness limitation for noncompact sections given in the fourth column of Table 5.1, the allowable bending stress may be calculated from (ASD F2.2):

$$F_b = [1.075 - 0.005(b_f / 2t_f)\sqrt{F_y}\,]F_y \tag{5.25}$$

5.5.2.3 Sections with slender compression elements

For I sections, channels, and tees, Eq. (5.19) shall be used for determining the allowable bending stress.

5.6 ALLOWABLE SHEAR STRESS ACCORDING TO THE ASD CODE

The allowable shear stress for hot-rolled sections is given by (ASD F4).

$$F_v = 0.40F_y \tag{5.26}$$

For very thin webs transverse stiffeners may be required. In this case design of beams will be similar to design of a plate girder. Design of plate girders is covered in Chapter 9. The allowable shear stress for stiffened and unstiffened plate girders is given by Eq. (9.6). This equation at the limit of F_v = 0.4F_y yields C_v = 1.156. When the spacing of the stiffeners is very large, the coefficient k in Eq. (9.8) becomes k = 5.34. From Eq. (9.7) the limiting h/t_w ratio becomes $380/\sqrt{F_y}$. Therefore, in order to use the full allowable shear stress value F_v = 0.4F_y, we must have

$$\frac{h}{t_w} \leq \frac{380}{\sqrt{F_y}}$$

If this condition is not satisfied, the allowable shear stress will be less than 0.4F_y and shall be found from Eq. (9.6).

5.7 LOCAL WEB YIELDING ACCORDING TO THE ASD CODE

At the support and the points of concentrated loads, the web behaves like a short column, and the beam must transfer compression from the wide flange to the narrow web. When the magnitude of the

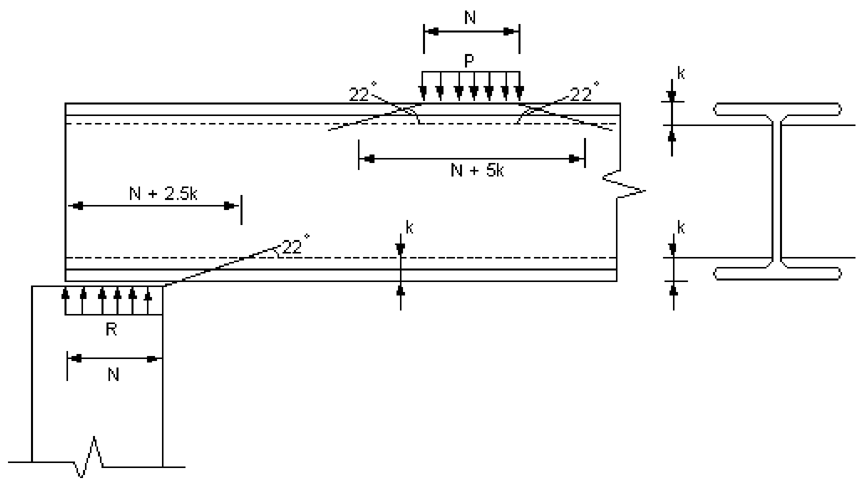


Figure 5.10 Load distribution for local web yielding

reaction or concentrated load is excessive, high compressive stresses may cause yielding at the junction of the web and the flange.

According to ASD K1.3, it is assumed that the load is distributed at 22-degree angles into the critical section at the toe of the fillet at a distance *k* from the face of the beam (Fig 5.10). (Experiments indicate that the load is in fact distributed over a length of $N + 5k$ to $N + 7k$.)

The compressive stress f_c at the web toe of the fillets due to concentrated loads shall satisfy the following equations (Fig. 5.10):

For interior loads:

$$f_c = \frac{P}{t_w(N+5k)} \le 0.66F_y \tag{5.27}$$

For end reactions:

$$f_c = \frac{P}{t_w(N + 2.5k)} \le 0.66F_y \qquad (5.28)$$

where, as indicated in Fig. 5.10.

P = concentrated load
R = reaction
N = bearing length (at least equal to k for end reactions)
k = distance from outer face of the flange to the web toe of fillet

When the compressive stress exceeds the allowable value of $0.66F_y$, the following solutions may be adopted.

1. Increase the bearing length
2. Use web stiffeners

5.8 DEFLECTIONS ACCORDING TO THE ASD CODE

Deflections of beams are sometimes limited to satisfy the aesthetic or comfort requirements or to prevent damage to nonstructural elements. For example, in beams supporting plastered ceiling in order to prevent cracking of plaster, the maximum deflection due to live load is limited to 1/360 of the span (ASD L3.1).

Deflection limitations are usually a matter of designer's judgement, depending on the type of the structure, nonstructural materials, and loading. As a general guideline, ASD commentary recommends the following rules (ASD commentary L3.1).

1. Depth of fully stressed beams should possibly be at least equal to F_y/800 times the span, where F_y is in ksi. If the depth of the member is less than this value, the allowable bending stress should be decreased in the same proportion as the depth is decreased from the recommended value.
2. Depth of fully stressed purlins in sloping roofs should be at least equal to F_y /1000 times the span, where F_y is in ksi.

5.9 EXAMPLES OF DESIGN OF BEAMS ACCORDING TO THE ASD CODE

Example 1

Portion *AB* of a continuous beam is made of a W30 x 108 section and a steel with yield stress of F_y = 65 ksi. It has the bending moment diagram shown in Fig. 5.11. Assuming lateral supports at *A* and *B* only, determine the maximum bending moment, M_{max}, this portion of the beam can support. Neglect the weight of the beam.

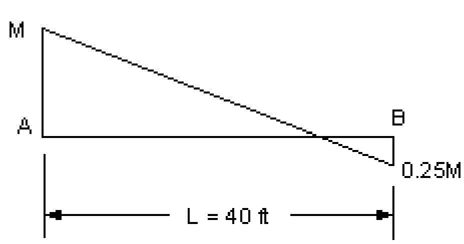


Figure 5.11

Solution

Properties of the W30 x 108 from the ASD manual:

d = 29.83 in.	S_x = 299 in.^{3}	t_w = 0.545 in.
b_f = 10.475 in.	t_f = 0.760 in.	r_T = 2.61 in.
d/A_f = 3.75 1/in.	$b_f/2t_f$ = 6.89	

We should first determine whether $F_b = 0.66F_y$ or not. We had better start with the requirements for unbraced length [Eqs. (5.9) and (5.10)] and then check the other requirements.

Unbraced length = L_u = 40(12) = 480 in.

Check Eq. (5.9):

$$\frac{76b_f}{\sqrt{F_y}} = \frac{76(10.475)}{\sqrt{65}} = 98.7\text{ in.} < 480\text{ in.} \qquad \therefore F_b \le 0.60F_y$$

$$\frac{b_f}{2t_f} = 6.89 < \frac{95}{\sqrt{F_y}} = \frac{95}{\sqrt{36}} = 11.8$$

$$h = d - 2t_f = 29.83 - 2(0.76) = 28.31\text{ in.}$$

$$\frac{h}{t_w} = \frac{28.31}{0.545} = 51.9 < \frac{640}{\sqrt{F_y}} = \frac{640}{\sqrt{36}} = 106.6$$

The section is not a section with slender compression elements.

Now, we can calculate the allowable bending stress, which is the larger value computed form Eqs. (5.15) and (5.16).

$$\frac{L_u}{r_T} = \frac{480}{2.61} = 183.9; \qquad \frac{M_1}{M_2} = 0.25\text{ (reverse curvature)}$$

$$C_b = 1.75 + 1.05(M_1 / M_2) + 0.3(M_1 / M_2)^2$$
$$= 1.75 + 1.05(0.25) + 0.3(0.25)^2 = 2.03 < 2.3$$

$$F_b^{''} = \frac{12{,}000 C_b}{L_u d / A_f} = \frac{(12{,}000)(2.03)}{(480)(3.75)} = 13.53 \text{ ksi} < 0.60 F_y = 39 \text{ ksi}$$

$$\sqrt{\frac{510{,}000 C_b}{F_y}} = \sqrt{\frac{510{,}000(2.03)}{65}} = 126.2 < \frac{L_u}{r_T}$$

$$F_b^{'} = \frac{170{,}000 C_b}{(L_u / r_T)^2} = \frac{170{,}000(2.03)}{(183.9)^2} = 10.20 \text{ ksi} < F_b^{''}$$

$$\text{Allowable bending stress} = F_b = 13.53 \text{ ksi}$$
$$M_{\max} = S_x F_b = (299)(13.53)/12 = 337.12 \text{ K - ft.}$$

To speed up the design process, as a general guideline, it is expedient to find the allowable bending stress first from Eq. (5.16). If this value happens to be greater than $0.60F_y$, the allowable bending stress is in fact $0.60F_y$, and there is no need to check the more involved Eq. (5.15).

Example 2

Select the lightest W section available in the ASD manual for the beam ABCDE shown in Fig. 5.12 using A36 steel. Lateral supports are provided at points *B* and *D* specified on the figure. The weight of the beam is included in the distributed load of 1 K/ft. Consider bending only.

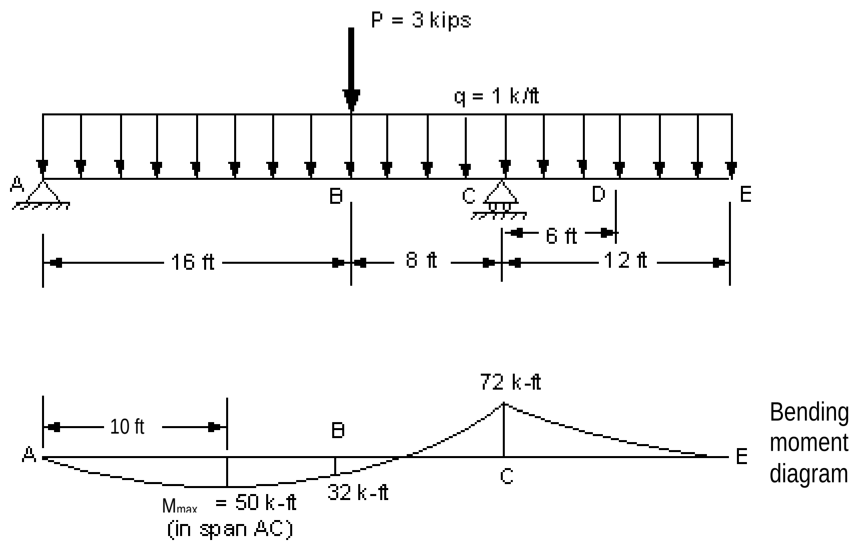


Figure 5.12

Solution

To start the iterative design process, initially assume the allowable bending stress to be F_b = 0.60F_y. (An experienced designer will most probably assume a somewhat lesser value because of the relatively long unbraced lengths.)

Maximum bending moment along the beam:

$$M_{\max} = (72)(12) = 864 \text{ K - in.}$$

$$S_x = \frac{M_{\max}}{F_b} = \frac{864}{0.6(36)} = 40 \text{ in.}^3$$

Try W16 x 31.

S_x = 47.2 in.^{3} d/A_f = 6.53 1/in. r_T = 1.39 in. b_f = 5.525 in.

a. Check for lateral support from *A* to *B*.

$$L_u = 192 \text{ in.}$$
$$C_b = 1$$

(The bending moment at a point within the unbraced length is larger than at both ends.)
Check whether $F_b = 0.66F_y$. From Eq. (5.9):

$$\frac{76b_f}{\sqrt{F_y}} = \frac{76(5.525)}{\sqrt{36}} = 70 \text{ in.} < L_u \quad \therefore F_b \le 0.60F_y$$

$$\sqrt{\frac{102{,}000C_b}{F_y}} = 53; \qquad \sqrt{\frac{510{,}000C_b}{F_y}} = 119$$

$$\frac{L_u}{r_T} = \frac{192}{1.39} = 138.1 > \sqrt{\frac{510{,}000C_b}{F_y}}$$

$$F_b'' = \frac{12{,}000C_b}{L_u d / A_f} = \frac{(12{,}000)}{(192)(6.53)} = 9.57 \text{ ksi}$$

$$F_b' = \frac{170{,}000C_b}{(L_u / r_T)^2} = \frac{170{,}000}{(138.1)^2} = 8.91 \text{ ksi}$$

$$\therefore F_b = 9.57 \text{ ksi.}$$

$$f_b = \frac{M_{\max}}{S_x} = \frac{600}{47.2} = 12.71 \text{ ksi} > F_b$$ **N.G.**

Try W14 x 34.

S_x = 48.6 in.^{3} d/A_f = 4.56 1/in. r_T = 1.76 in. b_f = 6.745 in.

$$\frac{L_u}{r_T} = \frac{192}{1.79} = 109.1$$

$$F_b'' = \frac{12{,}000C_b}{L_u d / A_f} = \frac{(12{,}000)}{(192)(4.56)} = 13.71\,\text{ksi}$$

$$F_b' = \left[\frac{2}{3} - \frac{F_y (L_u / r_T)^2}{1{,}530{,}000C_b}\right]F_y = \left[\frac{2}{3} - \frac{36(109.1)^2}{1{,}530{,}000}\right](36) = 13.92\,\text{ksi}$$

$$F_b = 13.92\,\text{ksi.}$$

$$f_b = \frac{M_{\max}}{S_x} = \frac{600}{48.6} = 12.35\,\text{ksi} < F_b = 13.92\,\text{ksi}$$ **<u>O.K.</u>**

b. Check for lateral support from *B* to *C*

$$L_u = 8\,\text{ft} = 96\,\text{in.}$$

$$\frac{76b_f}{\sqrt{F_y}} = \frac{76(6.754)}{\sqrt{36}} = 85.4\,\text{in.} < 96\,\text{in.} \;\therefore F_b \le 0.60F_y$$

$$C_b = 1.75 + 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2$$

$$= 1.75 + 1.05\left(\tfrac{32}{72}\right) + 0.3\left(\tfrac{32}{72}\right)^2 = 2.27 < 2.3$$

$$F_b'' = \frac{12{,}000C_b}{L_u d / A_f} = \frac{(12{,}000)(2.27)}{(96)(4.56)} = 62.2 > 0.60F_y = 22\,\text{ksi}$$

$$\therefore F_b = 22\,\text{ksi.}$$

$$f_b = \frac{864}{48.6} = 17.78\,\text{ksi} < 22\,\text{ksi}$$ **O.K.**

c. Check for lateral support from *C* to *D*.

$$L_u = 6\,\text{ft} = 72\,\text{in.}$$

$$\frac{76 b_f}{\sqrt{F_y}} = \frac{76(6.745)}{\sqrt{36}} = 85.4\,\text{in.} > L_u$$

$$\frac{20{,}000}{(d / A_f) F_y} = \frac{20{,}000}{(4.56)(36)} = 121.8 > L_u$$

$$F_b = 0.66 F_y = 24\,\text{ksi.} > f_b = 17.78\,\text{ksi}$$ **O.K.**

USE W14 x 34

Note that for all W sections given in the ASD made of A36 steel except W6x15, the local flange and web buckling requirements for compactness are always satisfied.

Example 3

Select the lightest W16 for the cantilever beam shown in Fig. 5.13 using A36 steel with yield stress of 36 ksi. Load P_1 is applied in the vertical plane of symmetry. Load P_2 passes through the centroid of the section and makes an angle of 30 degree with the vertical plane of symmetry. (It is located in a plane perpendicular to the vertical plane of symmetry.)

Assume total lateral support and neglect the weight of the beam. Specify the point where the normal stress has the largest absolute value.

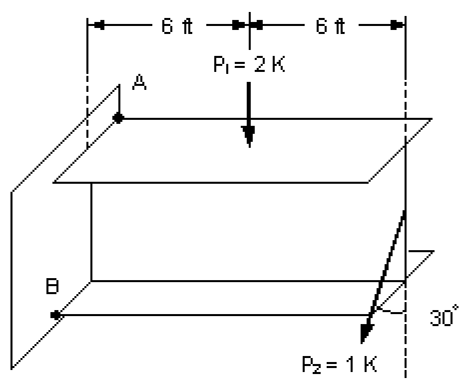


Figure 5.13

Solution

This beam is subjected to biaxial bending, that is, simultaneous bending about major and minor axes. If we denote these two axes by *x* and *y*, the resultant normal stress due to bending M_x about the *x*-axis and M_y about the *y*-axis for a doubly symmetric section is found from

$$f_b = \pm \frac{M_x}{S_x} \pm \frac{M_y}{S_y}$$

where S_x and S_y are section moduli with respect to *x* (major axis) and *y* (minor axis), respectively.

Denoting the maximum bending stress due to bending about the *x*-axis by $f_{bx} = M_x/S_x$ and the maximum bending stress due to bending about the *y*-axis by $f_{by} = M_y/S_y$ and noting that the allowable bending stresses about the major and minor axes are different, we must satisfy the following interaction equation for design of beams subjected to biaxial bending:

$$\frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \tag{5.29}$$

where F_{bx} and F_{by} are the allowable bending stresses with respect to major and minor axes, respectively. To start the iterative design process it may initially be assumed that $F_{bx} = 0.60F_y$ and $F_{by} = 0.75F_y$. Thus, the following approximate equation for the section modulus S_x may be used for preliminary selection of the section (depending on the span length, lateral bracing points, and the loading, a designer may choose other initial values for the allowable bending stresses):

$$S_x = \frac{M_x}{0.60F_y} + \frac{R_s M_y}{0.75F_y} \tag{5.30}$$

where $R_s = S_x/S_y$. This ratio can be estimated approximately for different rages of sections. The most frequent values for R_s are 7 for W27 to W36 sections, 5 to 6 for W16 to W24 sections, and 2 to 3 for W10 to W 14 sections.

For the example of Fig. 5.13, the maximum bending moment occurs at the fixed support.

M_x = 2(6)+(1)cos 30° (12) = 22.39 K-ft = 268.71 K-in.
M_y = (1)cos 60° (12) =6 K-ft = 72 K-in.

Assume R_s = 5.

$$S_x = \frac{M_x}{0.60F_y} + \frac{R_s M_y}{0.75F_y} = \frac{268.71}{24} + \frac{5(72)}{27} = 24.53\,\text{in.}^3$$

Try W16 x 26.

S_x = 38.4 in.^{3} and S_y = 3.49 in.^{3}

The section is compact and F_{bx} = 0.66F_y = 24 ksi and F_{by} = 0.75F_y = 27 ksi.

$$\frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} = \frac{M_x}{24S_x} + \frac{M_y}{27S_y} = \frac{268.71}{24(38.4)} + \frac{72}{27(3.49)}$$
$$= 0.291 + 0.764 = 1.06 > 1.0$$ **N.G.**

Try W16 x 31.

S_x = 47.2 in.^{3} and S_y = 4.49 in.^{3}

$$\frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} = \frac{268.71}{24(47.2)} + \frac{72}{27(4.49)}$$
$$= 0.237 + 0.594 = 0.831 < 1.0$$ **O.K.**

USE W16 x 31

Points A and B identified on Fig 5.13 are the points of maximum stress.

5.10 LOAD AND RESISTANCE FACTOR DESIGN OF BEAMS

5.10.1 Design for Flexure

In design of beams according to the LRFD code, the following limit states are considered:

1. Lateral torsional buckling (LTB)
2. Flange local buckling (FLB)
3. Web local buckling (WLB)

Similar to Sec. 5.3, the beam sections are divided into three categories: compact section, noncompact sections, and sections with slender compression elements.

For a section to be considered as compact, the width-thickness of its compression elements must not be greater than the limiting width-thickness ratios λ_p given in Table 5.3. When the width-thickness ratio of any one of the compression elements is lager than λ_p it is considered noncompact. When the width-thickness ratio of a compression element in a noncompact section exceeds λ_r given in Table 5.3, the element is called a slender compression element and the corresponding section is classified as a section with slender compression elements. Note that in Table 5.3, b is equal to $b_f/2$ for I-sections but equal to b_f for channels and hollow rectangular sections. The limiting ratios in Table 5.3 are similar to those in Table 5.1; however, there are differences too.

According to the LRFD code, the flexural design strength is $\phi_b M_n$, where

ϕ_b = resistance factor for flexure = 0.9
M_n = nominal flexural strength

TABLE 5.3. LIMITING WIDTH-THICKNESS RATIOS FOR COMPRESSION ELEMENTS ACCORDING TO THE LRFD CODE (LIMIT STATES OF FLB AND WLB) (LRFD TABLE B5.1)

Type of elements	Width-thickness ratio (λ)	Limiting width-thickness ratios	
		λ_p	λ_r
Flanges of rolled I sections and channels in flexure	$\frac{b}{t}$	$\frac{65}{\sqrt{F_y}}$	$\frac{141}{\sqrt{F_y - 10}}$
Flanges of I sections and channels in pure compression	$\frac{b}{t}$	NA	$\frac{95}{\sqrt{F_y}}$
Webs of I sections, channels, and hollow rectangular sections in flexural compression	$\frac{h}{t_w}$	$\frac{640}{\sqrt{F_y}}$ [a]	$\frac{970}{\sqrt{F_y}}$ [b]
Uniformly compressed stiffened elements	$\frac{b}{t}$ or $\frac{h}{t_w}$	NA	$\frac{253}{\sqrt{F_y}}$
Stems of tees	$\frac{b}{t}$	NA	$\frac{127}{\sqrt{F_y}}$
Flanges of hollow rectangular sections	$\frac{b}{t}$	$\frac{190}{\sqrt{F_y}}$	$\frac{238}{\sqrt{F_y}}$
Hollow circular sections in axial compression	$\frac{D}{t}$	NA	$\frac{3300}{F_y}$
Hollow circular sections in flexure	$\frac{D}{t}$	$\frac{2070}{F_y}$	$\frac{8970}{F_y}$

[a] When in addition to bending, the section is also acted on by an axial compressive force, the limiting width-thickness ratio becomes

$$\frac{h}{t_w} \le \begin{cases} \frac{640}{\sqrt{F_y}}\left[1 - \frac{2.75P_u}{\phi_b P_y}\right] & \text{when } \frac{P_u}{\phi_b P_y} \le 0.125 \\ \frac{191}{\sqrt{F_y}}\left[2.33 - \frac{P_u}{\phi_b P_y}\right] \ge \frac{253}{\sqrt{F_y}} & \text{when } \frac{P_u}{\phi_b P_y} > 0.125 \end{cases}$$

where P_u is the required axial strength and $P_y = AF_y$ is the yield strength.

[b] For combined bending and axial force

$$\frac{h}{t_w} \le \frac{970}{\sqrt{F_y}}\left[1 - 0.74\frac{P_u}{\phi_b P_y}\right]$$

The nominal flexural strength is the smallest value computed based on the following limit states of LTB, FLB, and WLB (LRFD F1 and Appendix F1).

1. For $\lambda \le \lambda_p$

$$M_n = M_p \tag{5.31}$$

2. For $\lambda_p < \lambda \le \lambda_r$

For the limit state of LTB

$$M_n = C_b\left[M_p - (M_p - M_r)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right] \le M_p \tag{5.32}$$

For the limit state of FLB and WLB:

$$M_n = M_p - (M_p - M_r)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) \tag{5.33}$$

3. For $\lambda > \lambda_r$

For LTB and FLB:

$$M_n = M_{cr} = S_x F_{cr} \le M_p \tag{5.34}$$

For WLB:

For λ of the web > λ_r, design as a plate girder (see Chapter 9).

M_p = plastic moment = ZF_y
M_{cr} = buckling moment
M_r = limiting buckling moment, equal to M_{cr} when $\lambda = \lambda_r$
λ = controlling slenderness parameter
= minor axis slenderness ratio L_u/r_y for LTB
= flange width-thickness ratio b/t_f for FLB
= web depth-thickness ratio for h/t_w for WLB
λ_p = limiting slenderness parameter for compact sections (largest value of λ for which $M_n = M_p$)
λ_r = limiting slenderness parameter for sections with slender elements (largest value of λ for which buckling is inelastic)
F_{cr} = critical stress
S_x = section modulus
Z = plastic modulus
L_u = unbraced length
r_y = radius of gyration about minor axis

The quantity C_b is a factor to take into account the non-uniform bending moment distribution over an unbraced segment (LRFD F1.2a).

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} \tag{5.35}$$

M_A = absolute value of moment at quarter point of the unbraced segment

M_B = absolute value of moment at mid-point of the unbraced segment
M_C = absolute value of moment at three-quarter point of the unbraced segment
$M_{\max}$ = absolute value of maximum moment in the unbraced segment

The λ_p and λ_r values corresponding to FLB and WLB have been summarized in Table 5.3. The slenderness parameter λ_p corresponding to LTB for doubly symmetric I shapes bent about major axis is (LRFD F1.2a)

$$\lambda_p = 300/\sqrt{F_{yf}} \tag{5.36}$$

The slenderness parameter λ_r corresponding to LTB for I shapes bent about the major axis is (LRFD F1.2a)

$$\lambda_r = \frac{X_1}{F_L}\sqrt{1+\sqrt{1+X_2 F_L^2}} \tag{5.37}$$

where

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}} \tag{5.38}$$

$$X_2 = \frac{4C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2 \tag{5.39}$$

E = modulus of elasticity of steel (29,000 ksi)
G = shear modulus of steel (11,200 ksi)
A = cross-sectional area (in.^{2})
F_L = smaller of (F_{yf} - F_r) or F_{yw}
J = torsional constant (in.^{4})

C_w = warping constant (in.^{6}), $C_w \approx \frac{1}{4} I_y \left(d - t_f\right)^2$
I_y = moment of inertia about minor axis (in.^{4})

It should be noted that there are no LTB λ_p and λ_r limits for any I shape bent about its minor axis (LTB is not a problem in this case).

The critical stress for F_{cr} LTB for bending about the major axis is given by

$$F_{cr} = \frac{C_b X_1 \sqrt{2}}{\lambda} \sqrt{1 + \frac{X_1^2 X_2}{2\lambda^2}} \tag{5.40}$$

and for FLB of the rolled shapes is given by

$$F_{cr} = \frac{20{,}000}{\lambda^2} \tag{5.41}$$

When the bending is about the major axis, the limiting buckling moment M_r for LTB and FLB is given by

$$M_r = F_L S_x \tag{5.42}$$

and for WLB is given by

$$M_r = F_{yf} S_x \tag{5.43}$$

For bending about the minor axis, M_r needs to be evaluated for FLB only, and its value is specified by

$$M_r = F_y S_y \tag{5.44}$$

5.10.2 Design for Shear

The design shear strength of unstiffened webs is $\phi_b V_n$, where (LRFD F2)

ϕ_b = resistance factor for shear = 0.9
V_n = nominal shear strength

The nominal shear strength is determined as follows:

$$V_n = \begin{cases} 0.60 F_{yw} A_w & \dfrac{h}{t_w} \le \dfrac{418}{\sqrt{F_{yw}}} \\ 0.60 F_{yw} A_w \dfrac{\dfrac{418}{\sqrt{F_{yw}}}}{h/t_w} & \dfrac{418}{\sqrt{F_{yw}}} < \dfrac{h}{t_w} \le \dfrac{523}{\sqrt{F_{yw}}} \\ \dfrac{132{,}000 A_w}{(h/t_w)^2} & \dfrac{523}{\sqrt{F_{yw}}} < \dfrac{h}{t_w} \le 260 \end{cases} \qquad (5.45)$$

where

F_{yw} = yield stress of the web (ksi)
A_w = web area (in.^{2})

5.10.3 Design Consideration for Concentrated Loads

5.10.3.1 Local web yielding

Similar to the ASD requirements presented in section 5.7 and Figure 5.10, the design compressive strength of the web at the toe of the fillet under concentrated load is ϕR_n, where ϕ = 1.0 and R_n is computed as follows (LRFD K1.3):

a. When the concentrated load is applied to the interior of a span (that is, at a distance of at least the beam's depth from its end):

$$R_n = (5k + N)F_{yw}t_w \tag{5.46}$$

b. When the concentrated load is applied at or near the end or the member (that is, at a distance not more than the beam's depth from its end):

$$R_n = (2.5k + N)F_{yw}t_w \tag{5.47}$$

where

N = length of bearing
k = distance from the outer face of the flange to the web toe of fillet

Equation (5.46) and (5.47) can be used to find the minimum required bearing length. When the required strength exceeds ϕR_n, a pair of stiffeners or a doubler plate, extending at least one-half of the depth of the web shall be provided.

5.10.3.2 Web crippling

The design compressive strength for unstiffened portion of webs subjected to concentrated loads is ϕR_n, where ϕ = 0.75 and R_n is computed as follows (LRFD K1.4):

a. When the concentrated load is at a distance not less than $d/2$ from the end of the member (d is the overall depth of the cross section):

$$R_n = 135 t_w^2 \left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5} \right] \sqrt{F_{yw} t_f / t_w} \tag{5.48}$$

b. When the concentrated load is at a distance less than $d/2$ from the end of the member:

For $N/d \leq 0.2$,

$$R_n = 68 t_w^2 \left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5} \right] \sqrt{F_{yw} t_f / t_w} \tag{5.49}$$

For $N/d > 0.2$,

$$R_n = 68 t_w^2 \left[1 + \left(\frac{4N}{d} - 0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5} \right] \sqrt{F_{yw} t_f / t_w} \tag{5.50}$$

When the required strength exceeds ϕR_n, a transverse stiffener, a pair of stiffeners, or a doubler plate, extending at least one-half of the depth of the web shall be provided.

5.10.3.3. Sidesway web buckling

When flanges of a beam subjected to concentrated loads are not restrained against relative movement by stiffeners or lateral bracing, the design compressive strength is ϕR_n, where ϕ = 0.85 and R_n is computed as follows (LRFD K1.5):

a. When the compression flange is restrained against rotation and $(h/t_w)/(L_u/b_f) \le 2.3$:

$$R_n = \frac{C_r t_w^3 t_f}{h^2}\left[1 + 0.4\left(\frac{h/t_w}{L_u/b_f}\right)^3\right] \tag{5.51}$$

where

$$C_r = \begin{cases} 960{,}000 \text{ (ksi) when } M_u < M_y \text{ at the location of the concentrated force} \\ 480{,}000 \text{ (ksi) when } M_u \ge M_y \text{ at the location of the concentrated force} \end{cases}$$

When $(h/t_w)/(L_u/b_f) > 2.3$, this limit state does not have to be checked.

When the compression flange is not restrained against rotation and $(h/t_w)/(L_u/b_f) \le 1.7$:

$$R_n = \frac{C_r t_w^3 t_f}{h^2}\left[0.4\left(\frac{h/t_w}{L_u/b_f}\right)^3\right] \tag{5.52}$$

When $(h/t_w)/(L_u/b_f) > 1.7$, this limit state does not have to be checked.

5.10.3.4 Compression buckling of the web

When concentrated loads are applied to both flanges of the beam, the design compressive strength for unstiffened portions of webs is ϕR_n, where ϕ = 0.90 and

$$R_n = \frac{4{,}100 t_w^3 \sqrt{F_{yw}}}{h} \tag{5.53}$$

5.11 WEB-BASED INTERACTIVE DESIGN OF BEAMS

The applet for interactive design of simply-supported steel beams (including beams with left and right overhangs) consists of *Input* and *Results* panels shown in Figures 5.14 and 5.15, respectively. A separate configuration panel is displayed at the bottom of these panels. We have avoided using a toolbar and a scrollbar in order to reduce the screen size of the applet so that the users whose screen resolutions are set at less than 800X600 can still view the entire applet without scrolling. The user can toggle back and forth to different panels by clicking on the corresponding tab attached to them.

While the student is entering input values in the input panel the inputted values will be displayed immediately on the beam configuration panel. The locations of the supports and lateral bracing points are entered one at a time using the *Add* button. The values can be reset at anytime using the *Reset* button. The loading can consist of a distributed load of

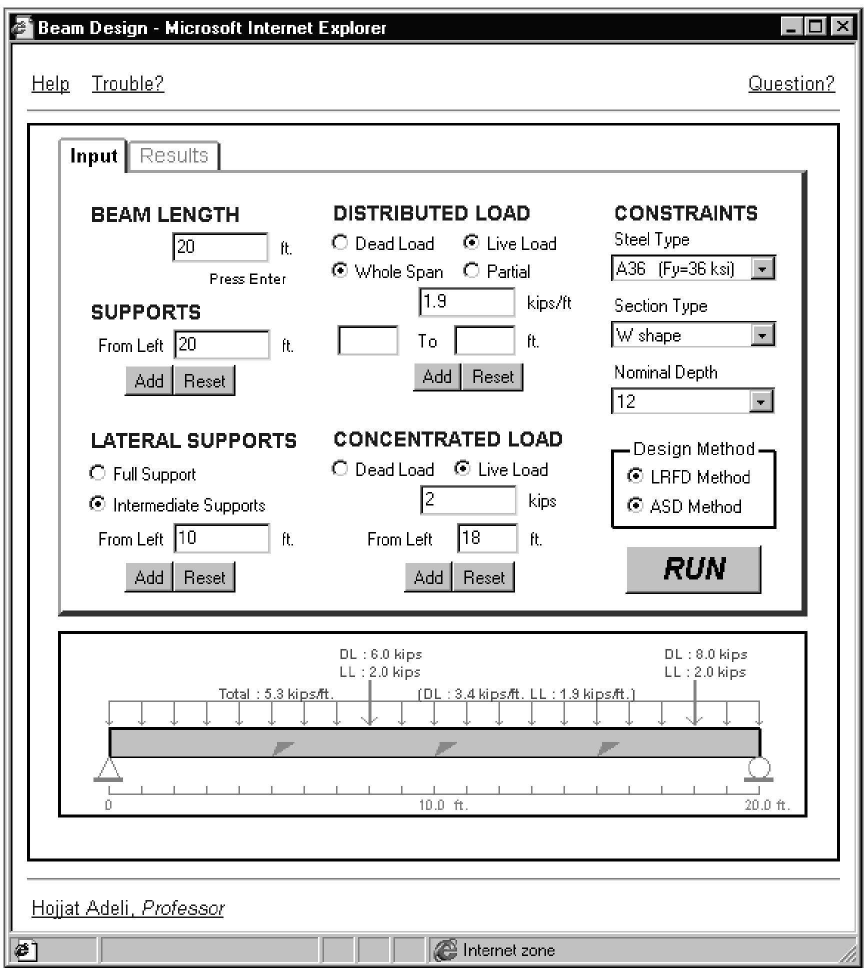


Figure 5.14 Input panel for the beam design applet

uniform intensity on the entire span or any portion or portions of the span and any number of concentrated loads. The basis of design can be AISC ASD (AISC, 1995) or AISC LRFD (AISC, 1998) specifications. For the latter it is necessary to input the dead and live load components separately.

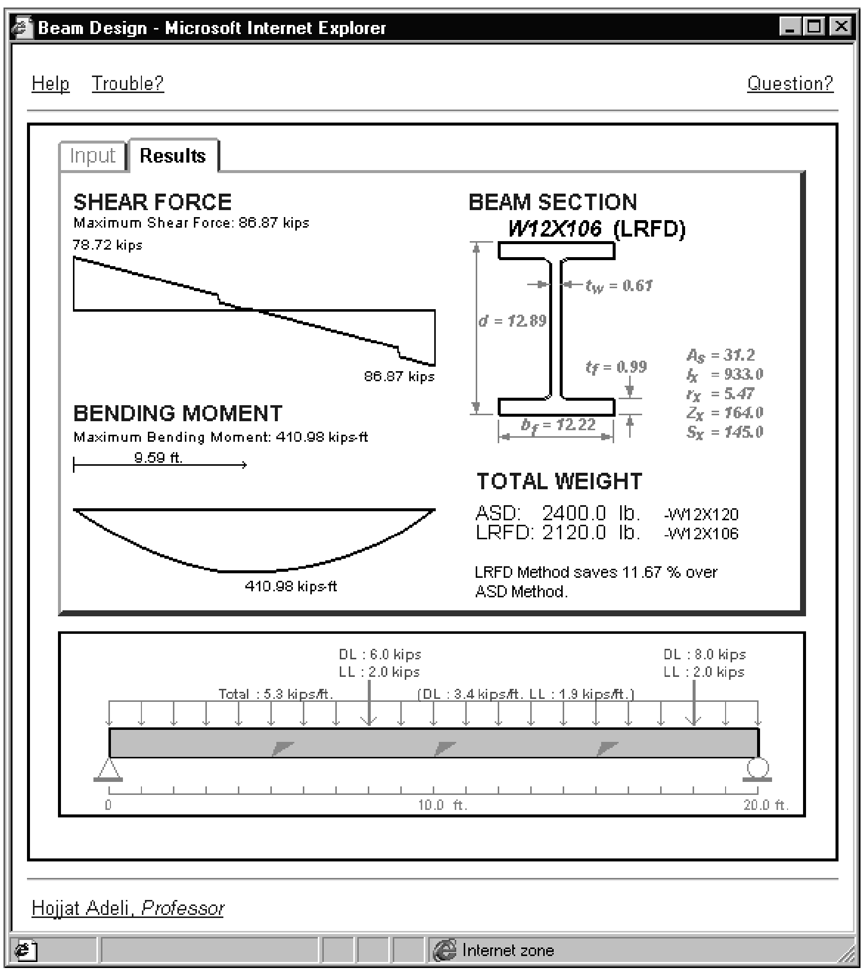


Figure 5.15 Results panel for the beam design applet

The student can choose the nominal depth of the section and ask the applet to find the lightest section with that nominal depth, or ask the applet to come up with the absolute lightest section irrespective of the nominal depth. The student can select any steel type provided in the AISC ASD or LRFD manuals. Section type can be W shape, M shape, S shape, HP shape, WT shape, C shape, or MC shape.

This applet as well as all other applets created for the course are intended to let the student perform the design with the minimum amount of data entry. Building upon the idea of the *Redesign Menus* presented in Adeli (1988), the user can perform redesigns repeatedly by simply changing only one or several of the input values. Based on the information obtained from the initial design, the user can modify or limit any design parameter and request a new design without the need to start all over again. This is an effective tutorial feature considering the open-ended nature of the design problems. The student can find the answers to a lot of what-if scenarios in a very short period of time. This not only boosts the learning experience tremendously but also makes it more interesting.

The results panel displays all the necessary and useful information including the shear diagram, the bending moment diagram, the magnitude of the maximum bending moment, the location of the maximum bending moment, and the cross-section of the selected member with all the dimensions for the selected design approach. Further, it will also show the answer based on the other design approach. In Figure 5.15, for example, the user has selected LRFD for the basis of design. But, the design on the basis of ASD is also presented as well as the relative magnitude of weight savings. The comparison of designs based on the ASD and LRFD codes enhances the design feel and experience of the student.

5.12 PROBLEMS

5.1 A W21x83 made of A36 steel with yield stress of 36 ksi is used as a simply supported beam with a span of 15 ft carrying a uniformly distributed load of intensity 5 Kips/ft including its own weight. The end supports make an inclination of 10 degrees with the horizontal, as shown in Figure 5.16. Check the adequacy of the beam according to the ASD code. Assume that the load acts through the centroid of the cross section. Lateral support is provided at the two ends of the beam only.

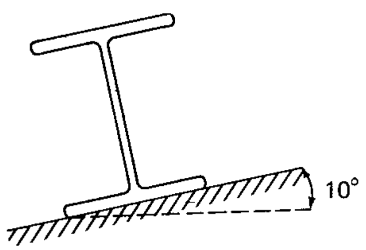


Figure 5.16

5.2 A structural tube is used as a beam to span a highway bridge of span length 32 ft. The beam is simply supported and subjected to a

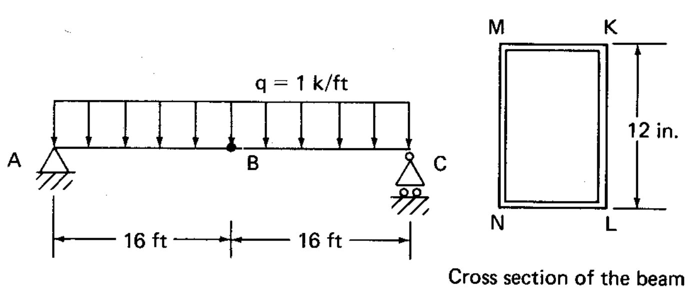


Figure 5.17

vertical (dead + live) load of 1 K/ft and a horizontal wind load of 60 psf acting on the face MN of the tube (Fig. 5.17). In addition to supports A and C, the beam has lateral support at the midpoint B. Using steel with yield stress of 50 ksi, select the lightest tube with a depth of 12 in.

5.3 A castellated beam has been built out of a W33x118 section as shown in Fig 5.18. If A36 steel is used (F_y = 36 ksi), what uniformly distributed load can this simply supported castellated beam carry for a span of 20 ft when the compression flange is braced laterally? You may not consider this beam as a compact section. How can we increase the loading capacity of this beam by only minor modifications?

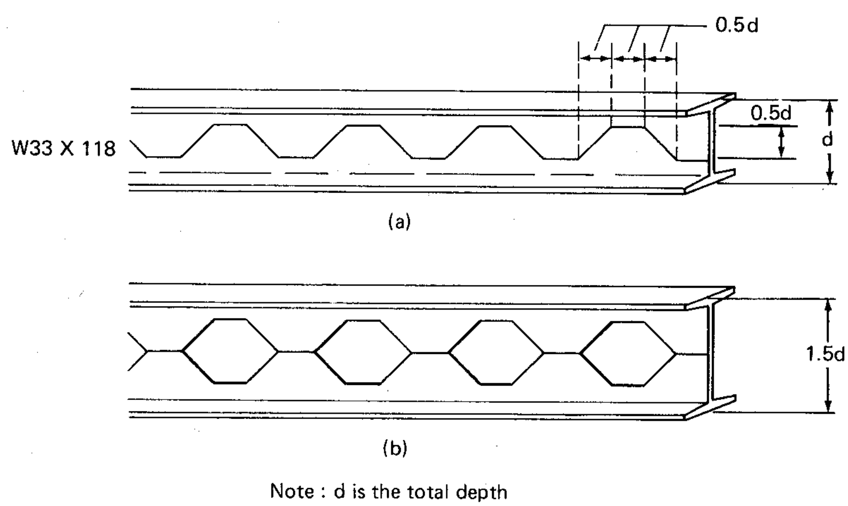


Figure 5.18

5.4 The floor of an indoor balcony consists of W beams of length L placed at spacing of 10 ft. Each beam is connected to the column at one end through a shear connection and is supported by a hanger at a distance x from the other end (Fig. 5.19). What should the distance x be in order to obtain the minimum weight beam? Using this distance, find the lightest W shape for the beam. Use A36 (F_y = 36 ksi) and assume total lateral support. Dead and live loads of the floor are 60 psf and 100 psf, respectively. L = 30 ft.

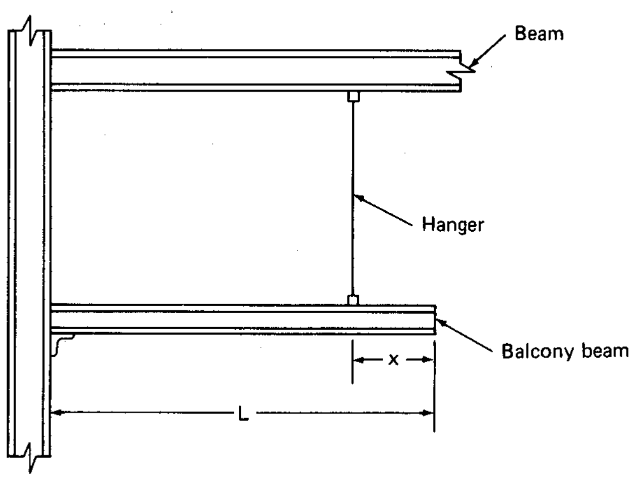


Figure 5.19

5.5 The floor arrangement shown in Fig 5.20 should be designed for a live load of 100 psf and the dead load of a 6-in. reinforced concrete slab and the beam weights. Design the typical beams in the N-S direction (identified by *F*, *G*, *H* and *I*) and the girders in the E-W direction (identified by *A*, *B*, *C*, *D* and *E*), assuming all of them are simply supported. Use 150 pcf for the weight of concrete, F_y = 36 ksi, and assume full lateral support.

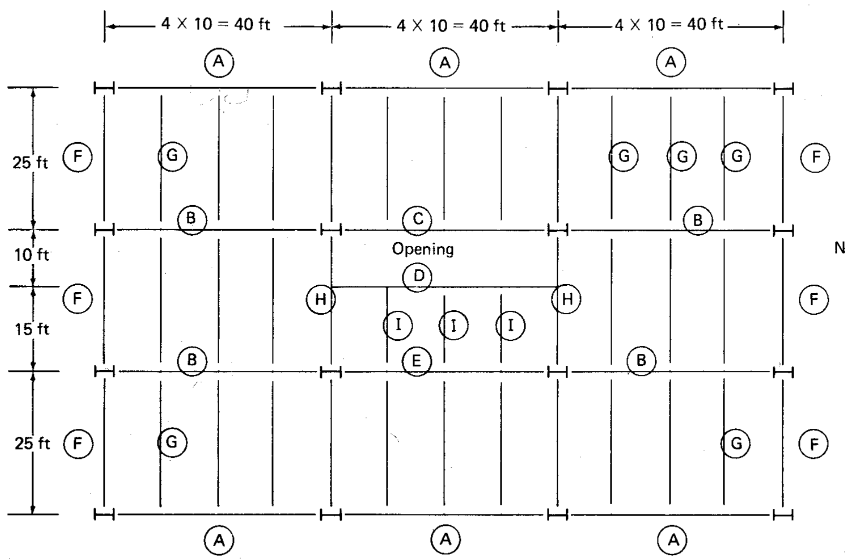


Figure 5.20

5.6 Solve problem 5.4, using steel with yield stress of 60 ksi.

5.7 The three-span continuous beam shown in Fig. 5.21 has a total length of L = 60 ft and is subjected to a distributed load of intensity q = 2 K/ft. Where should the internal supports be located (that is, find the distance x shown in the figure) in order to obtain the minimum

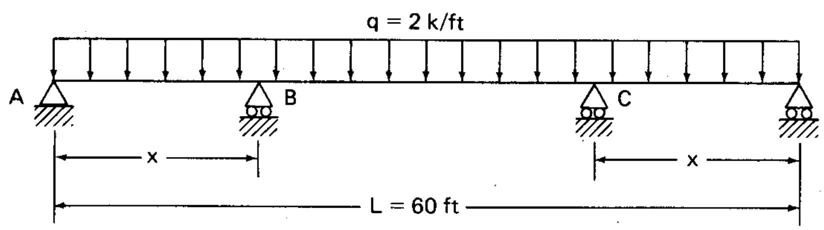


Figure 5.21

weight beam? Assume total lateral support. Using A36 steel with yield stress of 36 ksi, find the lightest W shape for the beam.

5.8 Solve Problem 5.7, using steel with yield stress of 65 ksi.

5.9 Find the lightest W shape for the beam shown in Fig. 5.22, using A36 steel (F_y = 36 ksi) and assuming

a. full lateral support
b. lateral supports at points A and B only

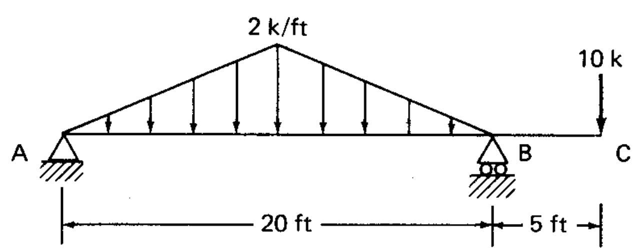


Figure 5.22

5.10 Solve Problem 5.9, using steel with yield stress of F_y = 50 ksi.

5.11 Find the lightest S shape for the beam shown in Fig. 5.22, using A36 steel (F_y = 36 ksi) assuming

a. full lateral support
b. lateral supports at points A and B only

5.12 Solve Problem 5.11, using A572 steel (F_y = 50 ksi).

5.13 Find the lightest double channels for the beam shown in Fig. 5.22, using A36 steel (F_y = 36 ksi) and assuming full lateral support.

5.14 Find the lightest rectangular tube for the beam shown in Fig. 5.22, using A572 steel with yield stress of 42 ksi and assuming

a. full lateral support

b. lateral supports at points A and B only

5.15 A built-up simply supported beam consists of a W24x68 and a C15x33.9 as shown in Fig. 5.23. The beam has a span of 30 ft and full support. Using A36 steel (F_y = 36 ksi), find the maximum uniformly distributed load this beam can carry.

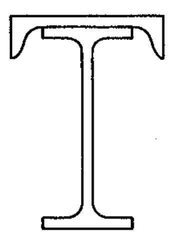
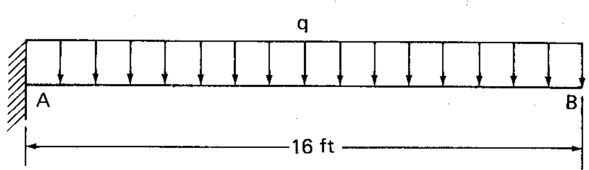


Figure 5.23

5.16 The cross section of the cantilever beam of Fig. 5.24 is shown in Fig. 5.25. Find the maximum load-carrying capacity of this beam (q_{max}), using A36 steel (F_y = 36 ksi) and assuming



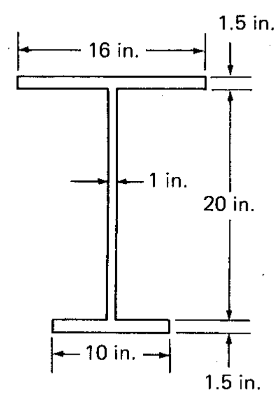


Figure 5.24 **Figure 5.25**

a. full lateral support
b. lateral supports at points A and B only

5.17 A simple beam with a span of 15 ft is subjected to two concentrated vertical loads, as shown in Fig. 5.26, and a uniformly distributed horizontal load of 0.5 K/ft. Find the lightest W12 section for the beam, using A36 steel (F_y = 36 ksi) and assuming

a. full lateral support
b. lateral supports at points A, B, C, and D only

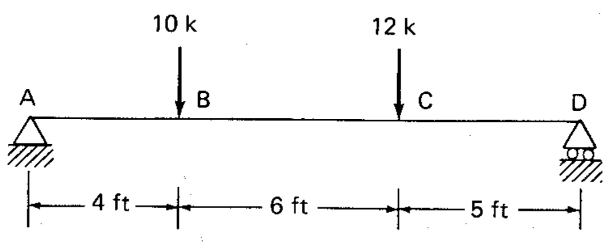


Figure 5.26

5.18 Solve the problem 5.17, substituting the uniformly distributed horizontal load by a concentrated horizontal load of magnitude 7.5 K applied at the middle point of the span. Assume that all the loads pass through the centroid of the section and thus produce no torsional effect.

5.19 In example 2 of this chapter, suppose a cover-plated W14 x 30 is used for the beam. What should be the size of the cover plates for the beam to carry the specified load? Assume that the cover plates are welded properly to the beam flanges.

5.20 Find the lightest W18 for Example 3 of this chapter.